Five Answers on Randomness

Jürgen Schmidhuber
IDSIA, Galleria 2, 6928 Manno-Lugano, Switzerland
University of Lugano, Switzerland
TU München, Germany
juergen@idsia.ch - http://www.idsia.ch/˜juergen

Abstract

Five brief and highly biased answers to five questions on randomness posed by Hector Zenil: Why were you initially drawn to the study of computation and randomness? What have we learned? What don't we know (yet)? What are the most important open problems? What are the prospects for progress?

1 Why were you initially drawn to the study of computation and randomness?

The topic is so all-encompassing and sexy. It helps to formalize the notions of Occam's razor and inductive inference [9, 36, 10, 37, 12, 19], which are at the heart of all inductive sciences. It is relevant not only for Artificial Intelligence [7, 30, 28, 33] and computer science but also for physics and philosophy [14, 18, 20]. Every scientist and philosopher should know about it. Even artists should, as there are complexity-based explanations of essential aspects of aesthetics and art [16, 15, 32].

2 What have we learned?

In the new millennium the study of computation and randomness, pioneered in the 1930s [5, 39, 8, 36, 9, 12], has brought substantial progress in the field of theoretically optimal algorithms for prediction, search, inductive inference based on Occam's razor, problem solving, decision making, and reinforcement learning in environments of a very general type [7, 23, 25, 26, 21, 30, 28, 33]. It led to asymptotically optimal universal program search techniques [6, 22, 33] for extremely broad classes of problems. Some of the results even provoke nontraditional predictions regarding the future of the universe [14, 18, 20, 29] based on Zuse's thesis [40, 41] of computable physics [14, 18, 19, 27]. The field also is relevant for art, and for clarifying what science and art have in common [16, 15, 17, 24, 31, 32].

3 What don't we know (yet)?

A lot. It is hard to write it all down, for two reasons: (1) lack of space. (2) We don't know what we don't know, otherwise we'd know, that is, we wouldn't not know.

4 What are the most important open problems?

4.1 Constant resource bounds for optimal decision makers

The recent results on universal problem solvers living in unknown environments show how to solve arbitrary well-defined tasks in ways that are theoretically optimal in various senses, e.g., [7, 33]. But present universal approaches sweep under the carpet certain problem-independent constant slowdowns, burying them in the asymptotic notation of theoretical computer science. They leave open an essential remaining question: If an agent or decision maker can execute only a fixed number of computational instructions per unit time interval (say, 10 trillion elementary operations per second), what is the best way of using them to get as close as possible to the recent theoretical limits of universal AIs? Once we have settled this question there won't be much left to do for human scientists.

4.2 Digital physics

Another deep question: If our universe is computable [40, 41], and there is no evidence that it isn't [27], then which is the shortest algorithm that computes the entire history of our particular universe, without computing any other computable objects [14, 18, 29]? This can be viewed as the ultimate question of physics.

4.3 Coding theorems

Less essential open problems include the following. A previous paper [18, 19] introduced various generalizations of traditional computability, Solomonoff's algorithmic probability, Kolmogorov complexity, and Super-Omegas more random than Chaitin's Omega [2, 35, 1, 38], extending previous work on enumerable semimeasures by Levin, Gács, and others [42, 11, 3, 4, 12]. Under which conditions do such generalizations yield *coding theorems* stating that the probability of guessing any (possibly non-halting) program computing some object in the limit (according to various degrees of limit-computability [19]) is essentially the probability of guessing its shortest program [19, 13]?

4.4 Art & science

Recent work [24, 31, 32] pointed out that a surprisingly simple algorithmic principle based on the notions of data compression and data compression *progress* informally explains fundamental aspects of attention, novelty, surprise, interestingness, curiosity, creativity, subjective beauty, jokes, and science & art in general. The crucial ingredients of the corresponding *formal* framework are (1) a continually improving predictor or compressor of the continually growing sensory data history of the action-executing, learning agent, (2) a computable measure of the compressor's progress (to calculate intrinsic *curiosity* rewards), (3) a reward optimizer or reinforcement learner translating rewards into action sequences expected to maximize future reward. In this framework any observed data becomes temporarily interesting by itself to the self-improving, but computationally limited, subjective observer once he learns to predict or compress the data in a better way, thus making it subjectively simpler and more *beautiful*. Curiosity is the desire to create or discover more non-random, non-arbitrary, regular data that is novel and *surprising* not in the traditional sense of Boltzmann and Shannon [34] but in the sense that it allows for compression progress because its regularity was not yet known. This drive maximizes *interestingness*, the first derivative of subjective beauty or compressibility, that is, the steepness of the learning curve. From the perspective of this framework, scientists are very much like artists. Both actively select experiments in search for simple but new ways of compressing the resulting observation history. Both try to create new but non-random, non-arbitrary data with surprising, previously unknown regularities. For example, many physicists invent experiments to create data governed by previously unknown laws allowing to further compress the data. On the other hand, many artists combine well-known objects in a subjectively novel way such that the observer's subjective description of the result is shorter than the sum of the lengths of the descriptions of the parts, due to some previously unnoticed regularity shared by the parts (art as an eye-opener). Open question: which are practically feasible, reasonable choices for implementing (1-3) in curious robotic artists and scientists?

5 What are the prospects for progress?

Bright. Sure, the origins of the field date back to a human lifetime ago [5, 39, 8, 36, 9], and its development was not always rapid. But if the new millennium's progress bursts [30] are an indication of things to come, we should expect substantial achievements along the lines above in the near future.

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