

Multiscan association as a single-commodity flow optimization problem

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Abstract—Multiscan data association can significantly enhance tracking performance in critical radar surveillance scenarios involving multiple targets, low detection probability, high false alarm probability, evasive target maneuvers and finite radar resolution. Unfortunately, however, this approach is affected by the curse of dimensionality which hinders its real-time application for tracking problems with short scan periods and/or long association windows and/or many measurements. In this paper it is shown how the formulation of the multiscan association as a single commodity flow optimization problem allows a relaxation of the association problem which, on one hand, guarantees close-to-optimal association performance and, on the other hand, implies a significant reduction of the computational load.

I. INTRODUCTION

It is well known that in critical radar scenarios, like e.g. GMTI (Ground Moving Target Indicator) radar tracking, involving many targets with low detection probability moving in a highly cluttered environment, a considerable improvement in tracking performance can be achieved by using *multiscan*, in place of *single scan*, data association [1]. In fact, the memory of multiscan association allows the partial recovery of association errors which represent the main error source in multitarget tracking. On the other hand, the complexity of multiscan association grows exponentially with the size (number of scans) of the association window. More precisely let M be the number of measurements per scan, T the number of tracks and S the number of scans in the association window, then multiscan association amounts to a linear binary programming problem with “in the order of” TM^S variables, i.e. $O(TM^S)$, and $O(MS)$ constraints. To avoid the “curse of dimensionality”, Lagrangian relaxation techniques have been proposed [4]. In [3], a novel relaxation technique, by which it is possible to represent the association problem as a multi-commodity flow optimization problem on a suitable graph involving $O(MS)$ nodes and $O(M^2TS)$ arcs, has been proposed. This solution, quite efficient in practice, does not completely avoid the curse of dimensionality as it involves the solution of a linear binary programming problem for which the integrality property of the solution is not guaranteed. In this paper, a step further is carried out by reformulating multiscan data association as a single-commodity flow optimization. This allows to solve the problem, though in an approximate and sub-optimal way, in polynomial time. On the other hand,

simulation results obtained from realistic case studies will demonstrate the effectiveness of the proposed approximation.

II. FORMULATION OF THE MULTISCAN DATA ASSOCIATION PROBLEM

In this section, the multiscan association problem over a window of S scans (in short *S-D Assignment*) is described in detail. To this end, let T be the number of tracks¹. It is assumed that for each track $n = 1, 2, \dots, T$ an estimate $\bar{\mathbf{x}}_n$ of the track state at the beginning of the window is available. Further, for each $k = 1, 2, \dots, S$, let us denote by $\mathcal{Z}_k = \{\mathbf{z}_{1,k}, \mathbf{z}_{2,k}, \dots, \mathbf{z}_{M_k,k}\}$ the set of measurements obtained at scan k (M_k being the cardinality of \mathcal{Z}_k). Given the T tracks and the S sets of measurements \mathcal{Z}_k for $k = 1, 2, \dots, S$, the objective of *S-D Assignment* is to assign a sequence of S measurements to each track, where the k -th element of such a sequence is either taken from \mathcal{Z}_k or represents a missed detection.

Among all feasible assignments, an optimal one is found by minimizing a suitably defined cost. In this connection, let $c(m_1, m_2, \dots, m_S, n)$ denote the cost of associating a certain sequence (m_1, m_2, \dots, m_S) to the track $n = 1, 2, \dots, T$. Here, each variable m_k , for $k = 1, 2, \dots, S$, takes its value in the set $\{0, 1, \dots, M_k\}$ and refers either to the m_k -th measurement of the set \mathcal{Z}_k (when $m_k > 0$) or to a missed detection (when $m_k = 0$). The cost to be minimized is typically the total negative log-likelihood and takes the form

$$c(m_1, m_2, \dots, m_S, n) = \sum_{k=1}^S c_k(m_1, m_2, \dots, m_k, n)$$

where $c_k(m_1, m_2, \dots, m_k, n)$ is the cost of adding m_k to the partial sequence $(m_1, m_2, \dots, m_{k-1})$.

In order to derive an expression for each $c_k(m_1, m_2, \dots, m_k, n)$, some preliminary definitions are needed. Given a measurement \mathbf{z} and a state \mathbf{x} , let $\Lambda(\mathbf{z}, \mathbf{x})$ be the likelihood that the measurement \mathbf{z} originates from a target with state \mathbf{x} . Further, consider some filtering mechanism (e.g. the extended Kalman filter or a Sequential Monte Carlo filter)

¹In this paper, for the sake of brevity, the track deletion and track initialization problems are not considered. However, many solutions to these problems are available in the literature; see, e.g., [1] and the references therein.

that provides an estimate \mathbf{x} as a function of a measurement \mathbf{z} and an one-step-behind estimate \mathbf{x}^- . The filter is supposed to consist of two parts: a prediction step

$$\mathbf{x}^+ = \text{pred}(\mathbf{x}^-)$$

and an innovation update

$$\mathbf{x} = \text{update}(\mathbf{x}^+, \mathbf{z}).$$

The propagation of the other statistics (e.g., the covariance matrices for the extended Kalman filter) as well as their involvement in the computation of the estimate \mathbf{x} is omitted for the sake of compactness.

Then, one can write

$$c_k(m_1, m_2, \dots, m_k, n) = \begin{cases} -\log(P_d \Lambda(\mathbf{z}_{m_k, k}, \bar{\mathbf{x}}_{n, k})), & m_k > 0 \\ -\log(1 - P_d), & m_k = 0 \end{cases} \quad (\text{II.1})$$

where P_d is the detection probability and $\bar{\mathbf{x}}_{n, k}$ is the prediction of the state of track n at scan k on the basis of the partial sequence (m_1, \dots, m_{k-1}) . Given a sequence (m_1, m_2, \dots, m_S) , the predictions $\bar{\mathbf{x}}_{n, k}$ can be computed recursively as

$$\bar{\mathbf{x}}_{n, k} = \begin{cases} \text{pred}(\text{update}(\bar{\mathbf{x}}_{n, k-1}, \mathbf{z}_{m_{k-1}, k-1})), & m_{k-1} > 0 \\ \text{pred}(\bar{\mathbf{x}}_{n, k-1}), & m_{k-1} = 0 \end{cases}$$

for $k = 2, 3, \dots, S$. The recursion is initialized from

$$\bar{\mathbf{x}}_{n, 1} = \bar{\mathbf{x}}_n.$$

By exploiting the foregoing definitions, it is possible to give a mathematical formulation of *S-D Assignment*. With this respect, for any possible sequence of measurements (m_1, m_2, \dots, m_S) and for any track n , let us define a binary association variable $a(m_1, m_2, \dots, m_S, n)$ that takes value 1 if (m_1, m_2, \dots, m_S) is associated to track n and value 0 otherwise. Then, the optimal assignments can be obtained by minimizing the loss functional

$$\sum_{n=1}^T \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} \dots \sum_{m_S=0}^{M_S} c(m_1, m_2, \dots, m_S, n) \times a(m_1, m_2, \dots, m_S, n)$$

under the constraints

$$\sum_{n=1}^T \sum_{m_1=0}^{M_1} \dots \sum_{m_{k-1}=0}^{M_{k-1}} \sum_{m_{k+1}=0}^{M_{k+1}} \dots \sum_{m_S=0}^{M_S} a(m_1, \dots, m_{k-1}, m, m_{k+1}, \dots, m_S, n) \leq 1$$

for $m = 1, 2, \dots, M_k$, $k = 1, 2, \dots, S$ (II.2)

and

$$\sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} \dots \sum_{m_S=0}^{M_S} a(m_1, m_2, \dots, m_S, n) = 1$$

for $n = 1, 2, \dots, T$. (II.3)

Condition (II.2) is needed to ensure that each measurement can be assigned to at most one track. As to (II.3), it imposes that exactly one sequence be assigned to each track.

As should be evident, *S-D Assignment* turns out to be a linear binary programming problem with $O(TM^S)$ variables and $O(MS)$ constraints. Therefore, the complexity of the problem increases exponentially with the number of scans S (see also Fig. 1 where a graphical representation of the hypothesis tree for a single track is provided). Indeed, *S-D Assignment* has been shown to be NP-hard [4]. As a consequence, unless very small instances of the problem are considered, the possibility of solving it exactly is ruled out by the so-called curse of dimensionality (i.e., the exponential growth of the computational burden).

In [4], an efficient algorithm based on a successive Lagrangian relaxation technique was proposed for the approximate solution of *S-D Assignment*. More recently, in [3] we proposed a novel relaxation technique consisting of the reformulation of *S-D Assignment* as a multi-commodity flow optimization problem over a suitable graph involving $O(MS)$ nodes and $O(M^2TS)$ arcs: each track corresponds to a commodity and the minimum cost path needs to be determined for each commodity. Unfortunately, even such an approximate technique, while quite efficient in practice, does not completely avoid the curse of dimensionality, as it involves the solution of a linear binary programming problem that, in the worst case, requires an exponentially growing computational burden. To avoid this, in the next section, a different graph-based relaxation of the problem is provided that turns out to be more computationally efficient.

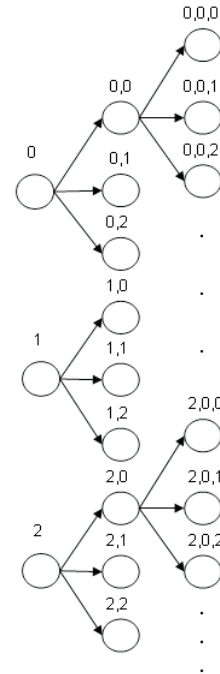


Fig. 1. Hypothesis tree for a single track.

III. S-D ASSIGNMENT AS MINIMUM COST FLOW

In this section, the key idea is to reformulate *S-D Assignment* as the problem of finding the minimum cost flow of a single commodity on a suitable graph. Specifically, a directed layered graph is considered so that the nodes are partitioned into $S + 1$ sets $\mathcal{L}_0, \mathcal{L}_1, \dots, \mathcal{L}_S$ (called layers) and all arcs connect consecutive layers. Let i_k denote the i -th node in layer \mathcal{L}_k of the graph. The graph is constructed as follows.

- Layer \mathcal{L}_0 consists of T nodes, one for each track, viz. $\mathcal{L}_0 = \{1_0, 2_0, \dots, T_0\}$.
- For $k = 1, 2, \dots, S$, layer \mathcal{L}_k consists of $T + M_k$ nodes where nodes $1_k, 2_k, \dots, T_k$ represent missed detections and each node $(T + i)_k$, for $i = 1, 2, \dots, M_k$, is associated with the i -th measurement of scan k . The need of considering T different missed detection nodes stems from the necessity of keeping the track paths separated.
- A destination node d is added so that all tracks can be thought as ending in d .

It is immediate to see that the graph resulting from the above described procedure involves $O((M + T)S)$ nodes and $O((M + T)^2 S)$ arcs. For the reader's convenience, such a graph is depicted in Fig. 2 for the case $S = 3$, $T = 2$, and $M_k = 2$ for $k = 1, 2, 3$.

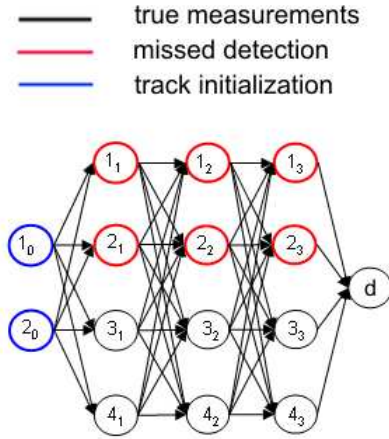


Fig. 2. Illustration of the layered graph associated with the approximate solution of *S-D Assignment*.

In this way, a sequence assigned to track n can be viewed as a path joining node n_0 to node d through S (measurement) nodes belonging to layers \mathcal{L}_k , $k = 1, 2, \dots, S$. Further, the *S-D Assignment* problem can be cast as a minimum-cost path on the above described graph provided that the cost associated with a path is additive across the arcs in the path, i.e., the cost of assigning a certain sequence (m_1, m_2, \dots, m_S) to track n takes the form

$$c(m_1, m_2, \dots, m_S, n) = c(n, m_1) + \sum_{k=2}^S c(m_{k-1}, m_k). \quad (\text{III.1})$$

Unfortunately, the association costs (II.1) derived in Section II are not exactly of the form (III.1) due to the dependence

of the terms for $k \geq 2$ on the track n as well as on the previous subsequence $(m_1, m_2, \dots, m_{k-2})$. In this work, in order to overcome such a drawback, an iterative Markovian approximation is proposed for such terms. This corresponds to associating a cost $C(j_{k-1}, i_k)$ to each arc (j_{k-1}, i_k) of the graph.

Following Section II, the costs between layers \mathcal{L}_0 and \mathcal{L}_1 can be readily defined as

$$C(n_0, i_1) = \begin{cases} -\log(1 - P_d) & i \leq T, i = n \\ \infty & i \leq T, i \neq n \\ -\log(P_d \Lambda(\mathbf{z}_{i-T,1}, \bar{\mathbf{x}}_n)) & i > T \end{cases}$$

for $n = 1, 2, \dots, T$ and $i = 1, 2, \dots, T + M_1$. Notice that, for a missed measurement ($i \leq T$), the cost $C(n_0, i_1)$ is finite only if $i = n$; in this way a specific missed detection index is assigned to each track.

As to the determination of the approximated costs between layers \mathcal{L}_k and \mathcal{L}_{k+1} , with $k=1, \dots, S-1$, it is customary to associate to each node i_k , with $k = 1, 2, \dots, S$ and $i = 1, 2, \dots, T + M_k$, an estimate $\hat{\mathbf{x}}(i_k)$ corresponding to the *best partial path* ending in node i_k . For $k = 2, 3, \dots, S$ and $i = 1, 2, \dots, T + M_k$ such estimates are recursively obtained as

$$\hat{\mathbf{x}}(i_k) = \begin{cases} \text{pred}(\hat{\mathbf{x}}(j_{k-1}^*)) & i \leq T \\ \text{update}(\text{pred}(\hat{\mathbf{x}}(j_{k-1}^*)), \mathbf{z}_{i-T,k}) & i > T \end{cases}$$

where

$$j^* = \arg \min_{j \in \{1, 2, \dots, T + M_{k-1}\}} C(j_{k-1}, i_k).$$

The recursion is initialized at layer \mathcal{L}_1 as

$$\hat{\mathbf{x}}(i_1) = \begin{cases} \bar{\mathbf{x}}_{n^*} & i \leq T \\ \text{update}(\bar{\mathbf{x}}_{n^*}, \mathbf{z}_{i-T,1}) & i > T \end{cases}$$

where

$$n^* = \arg \min_{n \in \{1, 2, \dots, T\}} C(n_0, i_1).$$

Then, for $k = 2, 3, \dots, S$, $j = 1, 2, \dots, T$, and $i = 1, 2, \dots, T + M_k$, one can write

$$C(j_{k-1}, i_k) = \begin{cases} -\log(1 - P_d) & i \leq T, i = j \\ \infty & i \leq T, i \neq j \\ -\log(P_d \Lambda(\mathbf{z}_{i-T,k}, \text{pred}(\hat{\mathbf{x}}(j_{k-1})))) & i > T \end{cases}$$

In order to derive the costs $C(j_{k-1}, i_k)$ with $j = T + 1, T + 2, \dots, T + M_{k-1}$, one needs to consider the *locally optimal* missed detection indices defined as

$$l^*(j_{k-1}) = \arg \min_{l \in \{1, 2, \dots, T\}} C(l_{k-2}, j_{k-1}).$$

Thus, one has

$$C(j_{k-1}, i_k) = \begin{cases} -\log(1 - P_d) & i \leq T, i = l^*(j_{k-1}) \\ \infty & i \leq T, i \neq l^*(j_{k-1}) \\ -\log(P_d \Lambda(\mathbf{z}_{i-T,k}, \text{pred}(\hat{\mathbf{x}}(j_{k-1})))) & i > T \end{cases}$$

for $k = 2, 3, \dots, S$, $j = T + 1, T + 2, \dots, T + M_{k-1}$, and $i = 1, 2, \dots, T + M_k$. According to such a definition,

a node j_{k-1} associated with a *true measurement* may lead to only one missed detection node corresponding to the locally optimal missed detection index $l^*(j_{k-1})$. This is instrumental in favouring paths that choose the same index for all the missed detections, thereby hopefully avoiding *track swapping*.

Based on the foregoing definitions and approximations, one can formulate *S-D Assignment* as a minimum cost flow problem on the considered graph. With this respect, let us consider continuous scalar variables $f(j_{k-1}, i_k) \in [0, 1]$ representing the flows through each arc (j_{k-1}, i_k) of the graph. Then one can address the minimization of the total flow cost

$$\sum_{(j_{k-1}, i_k)} C(j_{k-1}, i_k) f(j_{k-1}, i_k)$$

under the constraints

$$\sum_{i=1}^{T+M_1} f(n_0, i_1) = 1, \quad \text{for } n = 1, 2, \dots, T \quad (\text{III.2})$$

$$\sum_{i=1}^{T+M_S} f(i_S, d) = T \quad (\text{III.3})$$

$$\sum_{l=1}^{T+M_{k+1}} f(i_k, l_{k+1}) - \sum_{j=1}^{T+M_{k-1}} f(j_{k-1}, i_k) = 0$$

$$\text{for } i = 1, \dots, T + M_k \text{ and } k = 1, \dots, S \quad (\text{III.4})$$

$$\sum_{j=1}^{T+M_{k-1}} f(j_{k-1}, i_k) \leq 1$$

$$\text{for } i = 1, \dots, T + M_k \text{ and } k = 1, \dots, S. \quad (\text{III.5})$$

Each node belonging to layer \mathcal{L}_0 is a source node with unitary supply (see Eq. (III.2)). Node d is the unique target node with demand T (see Eq. (III.3)). Condition (III.4) imposes the conservation of flow in all the remaining nodes of the graph. Finally, the capacity constraint (III.5) ensures that at most a unitary flow can go through each node of the graph (this serves the same purpose as condition (II.2) in the original S-D assignment problem).

It is important to remark that the minimum cost flow problem is known to have the integrality property [5]: if all supplies, demands, and capacities are integral, then the problem has always an integer optimal solution. This implies that, in the considered framework, deriving the flow of minimal cost amounts to finding T mutually disjoint paths with minimal cost, i.e., the “best” multiscan multitarget association.

For what concerns the determination of the solution, since minimum cost flow is a linear programming problem, it can be solved by the simplex algorithm. Further, specific algorithms exist running in strongly polynomial time (i.e., the running time is polynomial in the number of nodes and arcs and is independent of both costs and capacities). For instance, the algorithm of [6] has complexity $O(A \log N(A + N \log N))$ for a graph with N nodes and A arcs. Thus, for our layered graph, such an algorithm yields the T mutually disjoint paths with minimal cost in $O((M + T)^4 S^2 \log[(M + T)S])$ time. As a final remark, it is pointed out that the idea of modelling

multiscan multitarget association as a minimum cost flow problem dates back to [7]. The main novelties of the proposed approach concern: i) the construction of the graph based on a relaxation of the S-D Assignment problem (as defined in [4]); ii) the possibility of dealing with missed detections and clutter. Attempts in this direction are also in [8], [9] where a different graph-based relaxation of *S-D assignment* is proposed and a Viterbi-like algorithm is adopted for the determination of the multiscan multitarget association.

IV. NUMERICAL RESULTS

In this section, it is shown that the proposed graph-based approximation has negligible impact on performance by means of simulation experiments in critical multitarget radar tracking scenarios.

For each target, the true state at discrete time t is $\mathbf{x}(t) = [x(t), \dot{x}(t), y(t), \dot{y}(t)]'$, where $(x(t), y(t))$ provides the position and $(\dot{x}(t), \dot{y}(t))$ the velocity of the target in Cartesian coordinates. The target's motion is described by the constant velocity model:

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{w}(t)$$

$$\mathbf{A} = \begin{bmatrix} 1 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T_s \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} T_s^2/2 & 0 \\ T_s & 0 \\ 0 & T_s^2/2 \\ 0 & T_s \end{bmatrix}$$

where T_s is the sampling time and $\mathbf{w}(t)$ is a zero-mean white process noise with covariance matrix \mathbf{Q} .

It is assumed that the sensor is a radar providing measurements of range $r(t)$, range rate $\dot{r}(t)$ and azimuth $\theta(t)$. Then the measurement equation is given by

$$\mathbf{z}(t) = \mathbf{h}(\mathbf{x}(t)) + \mathbf{v}(t) \quad (\text{IV.1})$$

where $\mathbf{z}(t) = [r(t), \theta(t), \dot{r}(t)]'$ is the measurement vector at time t , the nonlinear function $\mathbf{h}(\cdot)$ is defined as

$$\mathbf{h}(\mathbf{x}) = \begin{bmatrix} \sqrt{x^2 + y^2} \\ \angle(x + iy) \\ \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} \end{bmatrix},$$

and $\mathbf{v}(t)$ is a zero-mean white measurement noise with covariance matrix $\mathbf{R} = \text{diag}\{\sigma_r^2, \sigma_\theta^2, \sigma_{\dot{r}}^2\}$.

In the multitarget context let $\mathbf{x}_n(t)$ denote the state of the n th target (track) at time t .

Given the solution of the *S-D Assignment Problem* at time t , the state estimate $\hat{\mathbf{x}}_n(t)$ is updated via EKF with the last measurement of the multiscan sequence associated to the n th target. The estimation error for the n th target is defined as the distance between the true Cartesian position and its estimate, i.e.

$$d_n(t) = \sqrt{(x_n(t) - \hat{x}_n(t))^2 + (y_n(t) - \hat{y}_n(t))^2} \quad (\text{IV.2})$$

Let us also introduce the binary variable $L_n(t)$ taking value 1 if the measurement associated to track n is the right one and value 0 otherwise. Monte Carlo simulations have been

carried out by randomly varying the noise realizations, the time location of missed detections and clutter. The performance indices $\bar{d}(t)$ and $\bar{L}(t)$, obtained by averaging the previously defined $d_n(t)$ and $L_n(t)$ over the various tracks and over 50 Monte Carlo runs, will be evaluated in order to compare the single-commodity association approach proposed in this paper with the ideal case (exact association and $P_d = 1$) as well as with the multi-commodity association presented in [3].

Scenario 1

Let us first consider the multitarget case-study depicted in figure 3. In this scenario, there are $T = 20$ targets with detection probability $P_d = 0.8$ and 20 uniformly distributed clutter measurements at every step t ; the sampling time is $T_s = 5$ s; the radar has 50 m range and 1° azimuth resolution. The measurement noise standard deviations are $\sigma_r = 15$ m, $\sigma_{\dot{r}} = 1$ m/s and $\sigma_\theta = 0.25^\circ$.

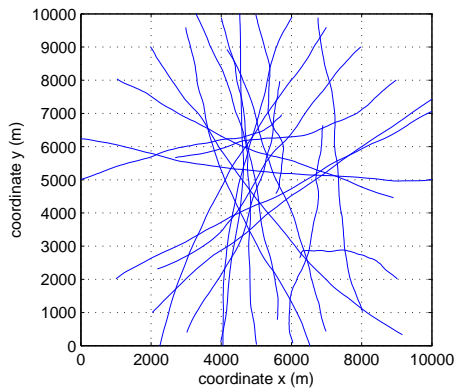


Fig. 3. First Scenario: target trajectories

Figure 4 reports the averaged distance $\bar{d}(t)$ obtained with the proposed *S-D Assignment* and in the ideal case. The percentage of correct assignments is shown in figure 5. From these figures, it is evident that the proposed single-commodity relaxation of *S-D Assignment* provides a negligible performance deterioration with respect to the ideal case.

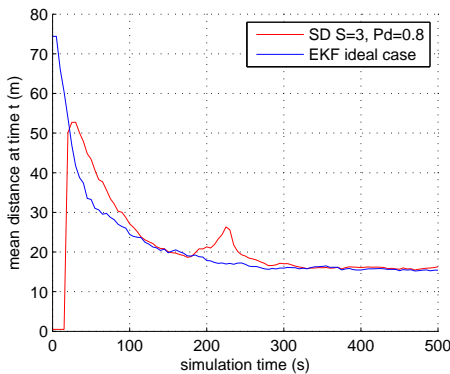


Fig. 4. Average target estimation error

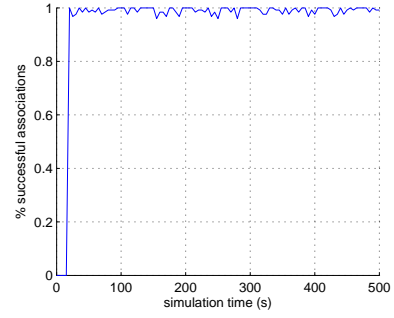


Fig. 5. Percentage of successful associations

Scenario 2

Let us now consider the multitarget case-study of figure 6 where three maneuvering targets are crossing at time $t = 160$ s and $t = 270$ s. The following parameters have been adopted in the simulation: radar scan time $T_s = 2$ s; range standard deviation $\sigma_r = 50$ m; range rate standard deviation $\sigma_{\dot{r}} = 1$ m/s; azimuth standard deviation $\sigma_\theta = 0.5^\circ$; detection probability $P_d = 0.8$; 20 clutter plots at each scan; radar resolution as in the previous scenario. Fig. 7 plots $\bar{d}(t)$ for the S-D single-commodity and S-D multi-commodity approaches as well as in the ideal case, while Fig. 8 reports $\bar{L}(t)$ for the S-D single-commodity and multi-commodity approaches. It can be seen that the performance of the single-commodity approach is very close to that of the multi-commodity approach in spite of a significantly reduced computational burden. Also notice that the performance of both (single-commodity and multi-commodity) associations are comparable with the performance achievable under ideal conditions with the only performance degradation in the time interval $200 \div 250$ s due to the targets' maneuvers and the fact that $P_d < 1$ rather than to association errors. For each target n , let $\hat{\mathbf{x}}_n^{EA}(t)$ and $\hat{\mathbf{x}}_n^{SD}(t)$ be the state estimates corresponding to the ideal case (exact association) and, respectively, to the approximate solution of the *SD-Assignment* problem. In order to compare the proposed association approach on flow optimization with the ideal case (exact association), the distance error

$$DE_n(t) = \sqrt{(\hat{x}_n^{EA}(t) - \hat{x}_n^{SD}(t))^2 + (\hat{y}_n^{EA}(t) - \hat{y}_n^{SD}(t))^2} \quad (IV.3)$$

has been evaluated. Monte Carlo simulations have been carried out by randomly varying the noise realizations, the time location of missed detections and clutter. Further, the following performance metrics have been taken into account: the mean distance error \overline{DE} , obtained by averaging the previously defined distance $DE_n(t)$ over time, over the various tracks and over 50 Monte Carlo runs, and the mean fraction of lost tracks \overline{LT} (a track is supposed to be lost by the association algorithm when the distance error $DE_n(t)$ exceeds a certain threshold).

Fig. 9 shows \overline{DE} and \overline{LT} obtained with the S-D multi-commodity approach for different radar scan times and reso-

lutions.

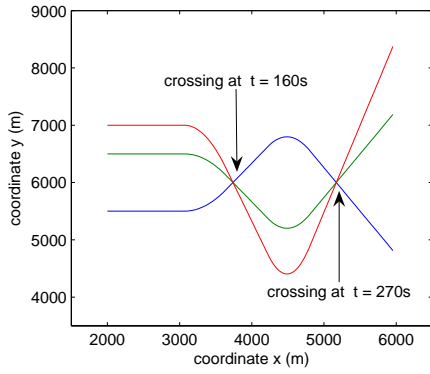


Fig. 6. Second Scenario: target trajectories

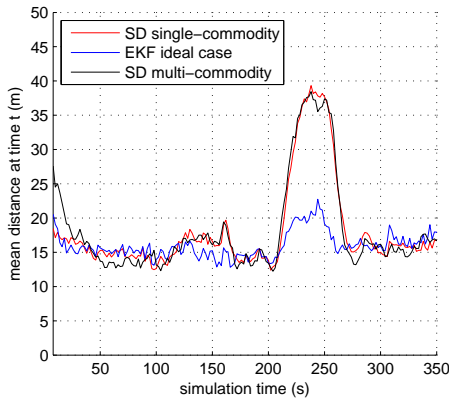


Fig. 7. Average target estimation error

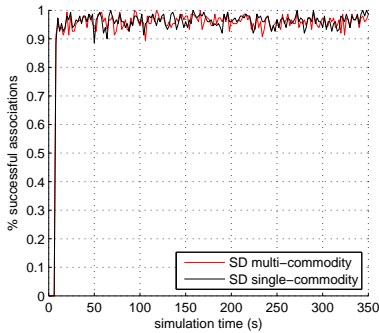


Fig. 8. Percentage of successful associations

V. CONCLUSIONS

A novel graph-based relaxation of the well known *S-D Assignment* problem has been proposed. Following previous work [3] in which the problem had been formulated as a multi-commodity flow optimization on a suitable graph, here

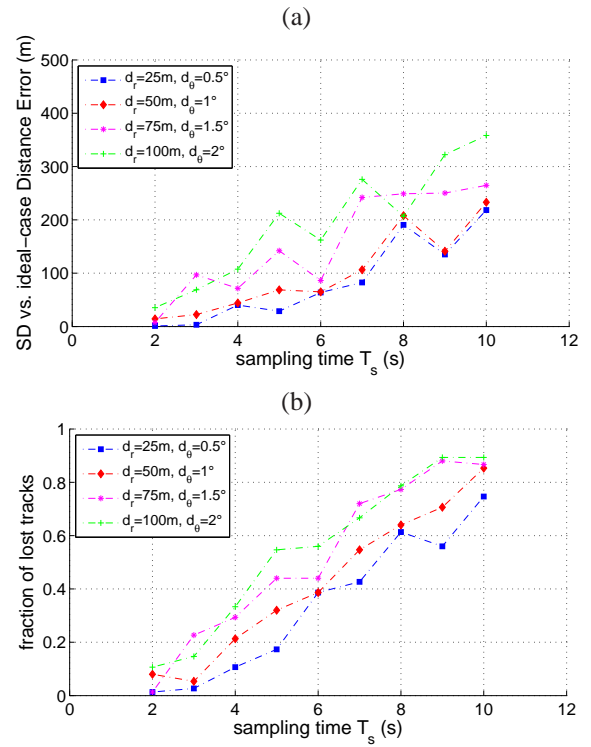


Fig. 9. Plots of (a) \overline{DE} and (b) \overline{LT} as functions of the radar scan time T_s for the single-commodity approach.

a computationally cheaper single-commodity approach has been presented. Simulation experiments in complex multitarget radar tracking scenarios have demonstrated the good association performance of the single-commodity relaxation scheme.

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