Interval Dominance based Data Association

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Abstract – *A new robust filtering method has recently been proposed based on closed-convex sets of probability distributions or, equivalently, coherent lower previsions, which are used to characterize uncertainty in the prior, likelihood and, respectively, state transition models. In this paper, we generalize this approach to the multi-target tracking problem by also addressing the uncertainty on the origin of the measurements (target or clutter). In particular, we show that this further source of uncertainty can be taken into account by using set of distributions and decision techniques for coherent lower previsions. Finally, we evaluate the performance of the proposed tracker by means of Monte Carlo simulations relative to difficult tracking scenarios such as manoeuvring and crossing targets.*

Keywords: coherent lower previsions, linear Gaussian vacuous mixture filter, data association.

1 Introduction

The objective of multi-target tracking is to estimate the state (e.g., position and speed) of targets moving in the surrounding environment from measurements collected by a sensor at each time step. This is a challenging problem for four different reasons: (a) target motion is (to some degree) unknown; (b) targets are not always detected by the sensor; (c) sensor generates a set of spurious measurements (clutter) not due to targets; (d) the source (either a target or clutter) originating a given measurement is unknown. A consequence of (b)–(d) is that the observation at each time step is a set of indistinguishable elements, only some of which are generated by detected targets.

The traditional approach to multi-target tracking consists of two steps: filtering and data association (DA). Filtering aims to estimate the state of each individual target. By defining a motion model for the target and measurement model for the sensor, a linear/nonlinear state space model is constructed. State estimation techniques for dynamic systems are then employed to estimate the target's state using sensor's measurements. The Kalman filter (and its variants) is one of the most known methods for (linear) state estimation. In the following, we call target's *track* the set of estimates of a given target returned by the filter at the various time instants.

Once filtering is performed for each target, DA techniques are employed to assign available measurements to the corresponding tracks. The main goal of DA is to distinguish which measurements are due to targets, which measurements due to clutter and, among the targets originated measurements, which is the measurement originated by a specific target. To reliably estimate the positions and motions of the objects despite the issues (a)–(d), many multi-target tracking algorithms, which combine filtering and DA, have been proposed (see [1] for a review). The robustness of multi-target tracking algorithms depends on their capacity to cope with all possible targets' behaviours. This is in general obtained by selecting a (finite) set of possible motion models and, then performing filtering and DA by taking account of the possible manoeuvres encoded by this set. This is for instance the case of Multiple Hypothesis Tracking (MHT) [5]. From a (Bayesian) probability point of view, set of models means finite set of probability distributions. In practice, for each model in the set, a (prior) distribution is used to address the uncertainty on the initial state of the target and another distribution to address the uncertainty on the time evolution of the state (state transition). However, there are many real cases where eliciting a finite set of probabilistic models, to cope effectively with all the target behaviours, may not be possible. In other cases, where it may be possible, the number of models in the set may be too large and, thus, increasing too much the computational time. In [2], a new robust filtering method has been proposed based on closed-convex sets of probability distributions used to characterize uncertainty in the prior, likelihood and, respectively, state transition models. The basic idea is that of solving the filtering problem without directly processing the distributions in the sets but propagating the lower and upper envelope of these sets of distributions. This is obtained by employing results from the theory of Coherent Lower Previsions (CLP) [6]. In the filtering context, where the quantity of interests are means, credible regions of the state etc., this means to propagate the lower and upper values of these functions. In [2], this new robust filtering approach has then been specialised to a special class of CLPs, called linear Gaussian-vacuous mixtures, which is the family of all convex combinations of a known nominal distribution (Gaussian) with arbitrary distributions. This family can be used to address estimation problems in which we take into account that our nominal model (linear Gaussian) can be inexact and, thus, we perturb (contaminate) it to reflect this modelling uncertainty. As it has been shown in [2], the resulting *linear Gaussian-vacuous mixtures* (LGVM) filter is naturally robust to unmodelled behaviours and, thus, is a perfect candidate to be used in multi-target tracking, where one of the main issue is to gain robustness w.r.t. to unmodelled manoeuvres. The aim of this paper is twofold.

First, we use the general results in [2] in order to address another source of uncertainty (apart from the motion models) which is always present in the multi-target tracking problem, i.e. the uncertainty on the origin of the measurements (target or clutter). We will show that this further source of uncertainty can be addressed by using set of distributions and decision techniques for CLPs. In particular, by using a decision criterion for CLPs known as interval dominance, we derive a new DA algorithm that addresses association in a similar (but more robust) way to Global Nearest Neighbour (GNN) [1], in the best case, and, to MHT, in the worst case. Second, we evaluate the performance of the multi-target tracking algorithm obtained by combining LGVM filter and this new DA technique by means of Monte Carlo simulation experiments, concerning critical multi-target tracking scenarios. We show that the proposed tracker is able to tackle with unmodelled behaviours of the targets in difficult environments (clutter presence, missed detections and interfering targets).

2 Coherent Lower Previsions

Let us briefly introduce the formalism of coherent lower previsions we shall use later in the paper. We refer to [6] for a detailed account of the theory and to [4] for a survey. Consider variables $Z_1, \dots, Z_n$, taking values in the sets $\mathcal{Z}_1, \dots, \mathcal{Z}_n$, respectively. For any subset $J \subseteq \{1, \dots, n\}$ we shall denote by Z_J the (new) variable $Z_J = (Z_j)_{j \in J}$, which takes values in the product space $\mathcal{Z}_J = \times_{j \in J} \mathcal{Z}_j$. We shall also use the notation $\mathcal{Z}^n$ for $\mathcal{Z}_{\{1,\dots,n\}}$. Notice that, with this notation, we can deal with both sets of variables or sets of vectors. A *gamble* f is a bounded real-valued function on $\mathcal{Z}_J$ for any J. The set of all gambles on $\mathcal{Z}_J$ is denoted by $\mathcal{L}(\mathcal{Z}_J)$. Intuitively, a gamble f is an "utility" function whose values depends on the a priori unknown value $Z_J = z_J$, i.e., if z_J turns out to be the true value of Z_J, we receive an amount $f(z_J)$ of utility. In the filtering problem, f can be the mean, variance etc. Consider now two disjoint subsets $O \neq \emptyset, U$ of $\{1, \dots, n\}$.

Definition 1. We call $\underline{E}_{Z_O}(\cdot|Z_U)$ a separately coherent conditional lower prevision on the set of gambles $\mathcal{L}(\mathcal{Z}_{O\cup U})$, if and only if for all $z_U \in \mathcal{Z}_U, f, g \in \mathcal{L}(\mathcal{Z}_{O\cup U})$, and $\lambda > 0$:

(SC1) $\underline{E}_{Z_O}(f|z_U) \geq \min_{z_O \in \mathcal{Z}_O} f$.

(SC2) $\underline{E}_{Z_O}(\lambda f|z_U) = \lambda \underline{E}_{Z_O}(f|z_U)$.

(SC3) $\underline{E}_{Z_O}(f+g|z_U) \geq \underline{E}_{Z_O}(f|z_U) + \underline{E}_{Z_O}(g|z_U)$.

If $U = \emptyset$, we obtain a (unconditional) coherent lower previsions $\underline{E}_{Z_O}(\cdot)$. ■

CLPs can be thought as lower expectations of utility functions. Because of lack of knowledge about the probability of the different $f(z_O)$, we may not be able to define precisely the expected utility for f, but only provide a lower bound for this expectation. This bound is our lower prevision for the gamble. A similar interpretation holds for the conditional case. The reason we use the terms previsions for expectations and gambles for utility functions is because the theory of CLPs is based on the subjective approach to probability [6]. We can also consider conditional *upper* previsions and we have $\overline{E}_{Z_O}(f|z_U) = -\underline{E}_{Z_O}(-f|z_U)$ for all gambles f. Upper previsions can be thought as upper expectations of utility functions.

Definition 2. A conditional lower prevision $\underline{E}_{Z_O}(\cdot|Z_U)$ on the set $\mathcal{L}(\mathcal{Z}_{O\cup U})$ is linear *if and only if it is separately coherent and $\underline{E}_{Z_O}(f+g|z_U) = \underline{E}_{Z_O}(f|z_U) + \underline{E}_{Z_O}(g|z_U)$ for all $z_U \in \mathcal{Z}_U$ and $f, g \in \mathcal{L}(\mathcal{Z}_{O\cup U})$.* ■

When a separately coherent conditional lower prevision $\underline{E}_{Z_O}(\cdot|Z_U)$ is linear, we denote it by $E_{Z_O}(\cdot|Z_U)$. Linear previsions correspond to the standard expectation operator $E_{Z_O}(\cdot|Z_U)$.

Theorem 1. A CLP $\underline{E}_{Z_O}(\cdot|Z_U)$ is separately coherent if and only if it is the lower envelope of a closed and convex set of conditional linear previsions, which we denote by $\mathcal{M}(\underline{E}_{Z_O}(\cdot|Z_U))$,[1] i.e., $\underline{E}_{Z_O}(f|z_U)$ is equal to:

$$\min\left\{E_{Z_O}(f|z_U) : E_{Z_O}(\cdot|z_U) \in \mathcal{M}(\underline{E}_{Z_O}(\cdot|z_U))\right\}. \quad (1)$$

■

From (1) and $\overline{E}_{Z_O}(f|z_U) = -\underline{E}_{Z_O}(-f|z_U)$, it follows that:

$$\underline{E}_{Z_O}(f|z_U) \leq E_{Z_O}(f|z_U) \leq \overline{E}_{Z_O}(f|z_U) \quad (2)$$

for any $E_{Z_O}(\cdot|z_U) \in \mathcal{M}(\underline{E}_{Z_O}(\cdot|z_U))$. Thus, specifying a CLP is equivalent to specify a closed and convex set of linear previsions (expectations) and viceversa. Hence, any closed and convex set of distributions can also be characterized by a CLP. Notice that,

[1] This is a bit of an abuse of notation, since actually for every $z_U \in \mathcal{Z}_U$ the set $\mathcal{M}(\underline{E}_{Z_O}(\cdot|z_U))$ is a set of linear previsions.

in the case $\underline{E}_{Z_O}(f|z_U) = \overline{E}_{Z_O}(f|z_U)$, i.e., the set $\mathcal{M}(\underline{E}_{Z_O}(\cdot|z_U))$ just includes a single linear prevision $E_{Z_O}(f|z_U)$, we reduce back to standard expectation or, equivalently, probability. Hence, we can see the classical expectation $E_{Z_O}(f|z_U)$ as the most informative CLP. The least informative CLP is the so called *vacuous prevision*, i.e., it is the CLP corresponding to the case $\mathcal{M}(\underline{E}_{Z_O}(\cdot|z_U))$ includes all the possible linear previsions. For vacuous previsions, it can be proved that: $\underline{E}_{Z_O}(f|z_U) = \min_{z_O \in \mathcal{K}_0} f(z_O)$, for any $\mathcal{K}_0 \subseteq \mathcal{Z}_0$[2], $f \in \mathcal{L}(\mathcal{Z}_{O \cup U})$ and $z_U \in \mathcal{Z}_U$.

As both linear and vacuous previsions are separately coherent, we can construct new CLPs by convex combination of the two [6, Ch. 2]. If $E^*_{Z_O}$ is a linear prevision, for each $0 \leq \epsilon \leq 1$, $\underline{E}_{Z_O}(fz_U) = \epsilon E^*_{Z_O}(f|z_U) + (1-\epsilon)\min_{z_O \in \mathcal{K}_0(z_U)} f(z_0)$ defines a new CLP which is called *linear-vacuous mixture*. This CLP is the lower envelope of the so called ϵ-contamination model [3], that is the class of all convex combination of $E^*_{Z_O}(\cdot|z_U)$ with an arbitrary linear prevision $E_{Z_O}(\cdot|z_U)$, i.e. $\mathcal{M}(\underline{E}_{Z_O}(f|z_U)) = \{\epsilon E^*_{Z_O}(f|z_U) + (1-\epsilon)E_{Z_O}(f|z_U) : \text{ for any } E_{Z_O}(f|z_U)\}$. In order to write down the solution of the filtering problem [2], we need three further results from Walley's theory: *marginal extension*, *generalized Bayes rule* and *epistemic irrelevance*.

Marginal extension is a generalization of the law of total probability (or chain rule) [6, Theorem 6.7.2]:

Definition 3. Let $\underline{E}_{Z_{O_1}}, \underline{E}_{Z_{O_2}}(\cdot|Z_{U_2}), \ldots, \underline{E}_{Z_{O_m}}(\cdot|Z_{U_m})$ be separately coherent conditional lower previsions with respective domains $\mathcal{L}(\mathcal{Z}_{O_1}), \mathcal{L}(\mathcal{Z}_{O_1 \cup U_1}), \ldots, \mathcal{L}(\mathcal{Z}_{O_m \cup U_m})$, where $U_1 = \emptyset$ and $U_j = \cup_{i=1}^{j-1}(U_i \cup O_i) = U_{j-1} \cup O_{j-1}$ for $j = 2, \ldots, m$. Their marginal extension *to $\mathcal{L}(\mathcal{Z}^n)$ is given by*

$$\underline{E}(f) = \underline{E}_{Z_{O_1}}(\underline{E}_{Z_{O_2}}(\ldots(\underline{E}_{Z_{O_m}}(f|Z_{U_m})|\ldots)|Z_{U_2})), \tag{3}$$

which is a CLP. ■

Generalized Bayes rule is a generalisation of Bayes'rule to CLPs [6, Ch. 6].

Definition 4. Let $\underline{E}_{Z_{O \cup U}}$ be a CLP, $f \in \underline{E}_{Z_O}(\cdot|Z_U)$ and $z_U \in \mathcal{Z}_U$. Under the hypothesis $\underline{E}_{Z_{O \cup U}}(I_{\{z_U\}}) > 0$[3], the GBR states that the separately conditional coherent lower prevision $\underline{E}_{Z_O}(f|z_U)$ is equal to the unique value of μ which solves:

$$\underline{E}_{Z_{O \cup U}}[I_{\{z_U\}}(f - \mu)] = 0. \tag{4}$$

where we use $I_{\{A\}}$ to denote the indicator function *of the set A, i.e., the function whose value is* 1 *for the elements of A and* 0 *elsewhere.* ■

This rule is a generalization to CLPs of Bayes rule from classical probability theory [6, Ch. 6]. In fact, if $\underline{E}_{Z_{O \cup U}}(I_{\{z_U\}}) > 0$ and we define $\underline{E}_{Z_O}(f|z_U)$ via the GBR, then it is the lower envelope of the conditional linear previsions $E_{Z_O}(f|z_U)$ that we can define using Bayes rule on the elements of $\mathcal{M}(\underline{E}_{Z_{O \cup U}})$. This can be verified by using the result (1). Finally, *epistemic irrelevance* generalizes to CLPs the notion of independence between variables [6, Sec. 9.1.1].[4]

Definition 5. Given the coherent lower prevision $\underline{E}_{Z_i}(\cdot|Z_j, Z_k)$, we say that Z_j is epistemically irrelevant *to Z_i conditional on Z_k if there is $\underline{E}_{Z_i}(\cdot|Z_k)$ such that $\underline{E}_{Z_i}(\cdot|Z_j, Z_k) = \underline{E}_{Z_i}(\cdot|Z_k)$.* ■

2.1 Decision making with CLPs

We conclude this section by discussing briefly the decision making approach for CLPs. The Bayesian methodology to decision making provides the action which maximises the expected value of some utility function (gamble). If $h_{a_i}(z_k)$ is the considered utility function, which depends on possible actions a_i and on the unknown value z_k of Z_k, then a_j is preferred to a_i [3] if and only if

$$E_{Z_k}(h_{a_i}) < E_{Z_k}(h_{a_j}). \tag{5}$$

When we consider CLPs, we deal with lower and upper previsions and, thus, with interval-valued expectations $[\underline{E}(\cdot), \overline{E}(\cdot)]$, leading to the problem of decision making under imprecision [6]. A consequence of imprecision is that, when the lower and upper expectations are substantially different, we may abandon the idea of choosing a unique action but returning a set of possible actions. There are various ways for decision making with CLPs such as: interval dominance, maximality, minimax etc [3],[6]. In the following, we consider *interval dominance* which is the weakest decision criterion for convex set of probabilities and, thus, the most robust. Under the interval dominance criterion, a_j dominates (is preferred to) a_i if and only if

$$\overline{E}_{Z_k}(h_{a_i}) < \underline{E}_{Z_k}(h_{a_j}). \tag{6}$$

Notice that, while for standard expectation the criterion (5) determines a total order between actions, (6) determines only a partial order. Thus, it can happen that (6) returns a set of actions that are undominated, i.e., we do not have any preference between actions in this set. A way to robustly tackle this case is to consider all consequences implied by these actions. This is the approach we will follow in Sec. 4.2 to address DA with CLPs.

3 CLPs based state estimation

In this Section, we review the main results stated in [2]. Assuming that the expectations (linear previsions) $E_{X_0}[\cdot]$ (prior), $E_{X_k}[\cdot|X_{k-1}]$ (state transition)

[2]Since $\underline{E}_{Z_O}(f|z_U)$ is a conditional CLP, the set $\mathcal{K}_0$ may depend on the value z_U.

[3]With $\underline{E}_{Z_{O \cup U}}(I_{\{z_U\}})$ we mean $\underline{E}_{Z_{O \cup U}}(I_{\{z_U \times \mathcal{Z}_O\}})$. This is an abuse of notation for the sake of simplicity.

[4]Notice that, within CLPs, there are other possible independence concepts. Epistemic irrelevance is the one used in this paper.

and $E_{Y_k}[\cdot|X_k]$ (likelihood) are known at each instant $k = 1, \ldots, t$, the aim of classical Bayesian state estimation is to derive the conditional expectation (linear prevision) $E_{X_t}[\cdot|Y^t = y^t]$, i.e., the conditional expectation of the state X_t given the sequence of measurements $\{Y_1 = y_1, Y_2 = y_2, \ldots, Y_t = y_t\}$. Hereafter we assume $X_k \in \mathcal{X}_k$ for each k, where $\mathcal{X}_k$ is a bounded subset of $\mathbb{R}^{n\times 1}$. We also regard the measurements $y_k \in \mathbb{R}^{r\times 1}$ for any $k \geq 1$ as idealisations of discrete events. This makes sense in practice because of the finite precision of the measurement instruments. We need this assumption later to apply GBR (4), see [2] for further details. In the following, we treat this aspect in a completely transparent way for the reader, assuming that the precision of the measurement instruments can be considered arbitrarily small. The conditional $E_{X_t}[\cdot|Y^t = y^t]$ can be computed recursively if we further assume that X_k is independent to X^{k-2} given X_{k-1} and that the measurement Y_t is independent to X^{k-1} and Y^{k-1} given X_k. Notice that, in Bayesian state estimation, propagating $E_{X_t}[\cdot|Y^t = y^t]$ up to time is equivalent to propagate the posterior probability density function (PDF) $p(x_t|y^t)$, which encodes the same information of $E_{X_t}[\cdot|Y^t = y^t]$. A particular case of Bayesian state estimation is when the state transition and measurement process are described by linear relationships:

$$\begin{cases} x_{t+1} &= Ax_t + w_t \\ y_t &= Cx_t + v_t, \end{cases} \tag{7}$$

with $w_t \sim \mathcal{N}(0, Q)$, $v_t \sim \mathcal{N}(0, R)$, $x_0 \sim \mathcal{N}(\hat{x}_0, P_0)$, and where the matrices A, C, Q, R are assumed to be known. Then the conditional PDF $p(x_t|y^t)$ is also Gaussian $\mathcal{N}(\hat{x}_t, P_t)$ where $\hat{x}_t = A\hat{x}_{t-1} + L_t[y_t - CA\hat{x}_{t-1}]$, $P_t = AP_{t-1}A_t^T + Q - L_t S_t L_t^T$, $S_t = C[AP_{t-1}A_t^T + Q]C_t^T + R$ and $L_t = [AP_{t-1}A_t^T + Q]C_t^T S_t^{-1}$. Here T denotes the transpose operator. These are the equations of Kalman filter (KF). Consider now the case in which the available information does not allow us to specify a unique probability measure describing each source of uncertainty in the dynamical system. We can then use coherent lower previsions to model the available knowledge. Consider CLPs $\underline{E}_{X_0}$, $\underline{E}_{X_k}[\cdot|X_{k-1}]$ and $\underline{E}_{Y_k}[\cdot|X_k]$ for $k = 1, \ldots, t$, and let us derive from them a separately coherent conditional lower prevision $\underline{E}_{X_t}[\cdot|y^t]$. According to GBR in Equation (4) in the case $Z_{O\cup U} = X^t \cup Y^t$, $g : \mathcal{X}_t \to \mathbb{R}$ and $y^t \in \mathcal{Y}^t$, the CLP $\underline{E}_{X_t}[g|y^t]$ can be obtained from the joint CLP $\underline{E}_{X^t,Y^t}[\cdot]$ by finding the value μ such that:

$$\underline{E}_{X^t,Y^t}[I_{\{y^t\}}(g - \mu)] = 0. \tag{8}$$

The unique value of μ which solves (8) corresponds to $\underline{E}_{X_t}[g|y^t]$, which is the solution of the state estimation problem in the case of CLPs. The following Lemma states how to build the joint $\underline{E}_{X^t,Y^t}[\cdot]$ from $\underline{E}_{X_0}$, $\underline{E}_{X_k}[\cdot|X_{k-1}]$ and $\underline{E}_{Y_k}[\cdot|X_k]$ for $k = 1, \ldots, t$.

Lemma 1. Consider the state vector $X_k \in \mathcal{X}_k$ and the measurements vector $Y_k \in \tilde{\mathcal{Y}}_k$ for each k and assume that the CLPs $\underline{E}_{X_0}$, $\underline{E}_{X_k}[\cdot|X_{k-1}]$ and $\underline{E}_{Y_k}[\cdot|X_k]$ are known for $k = 1, \ldots, t$. Furthermore, assume that, for each $k = 1, \ldots, t$, X^{k-2} and Y^{k-1} are epistemically irrelevant to X_k given X_{k-1} and that X^{k-1} and Y^{k-1} are irrelevant to Y_k given X_k, meaning that

$$\begin{aligned} \underline{E}_{X_k}[h_1|x^{k-1}, y^{k-1}] &= \underline{E}_{X_k}[h_1|x_{k-1}] \quad (9) \\ \underline{E}_{Y_k}[h_2|x^k, y^{k-1}] &= \underline{E}_{Y_k}[h_2|x_k]. \quad (10) \end{aligned}$$

$\forall h_1 \in \mathcal{L}(\mathcal{X}^k \times \mathcal{Y}^{k-1}), x^k, y^{k-1}$ and $\forall h_2 \in \mathcal{L}(\mathcal{X}^k \times \mathcal{Y}^k), x^k, y^{k-1}$.
Then, given the sequence of measurements $y^t = \{y_1, y_2, \ldots, y_t\}$, a gamble $g : \mathcal{X}_t \to \mathbb{R}$, and a constant $\mu \in \mathbb{R}$, it holds that:

$$\begin{aligned} &\underline{E}_{X^t,Y^t}[I_{\{y^t\}}(g - \mu)] = \underline{E}_{X_0}\Big[\underline{E}_{X_1}\Big[\underline{E}_{Y_1}\Big[\cdots \\ &\underline{E}_{X_t}\Big[\underline{E}_{Y_t}\Big[I_{\{y^t\}}(g - \mu)\Big|X_t,\Big]\Big|X_{t-1}\Big]\cdots\Big|X_1\Big]\Big|X_0\Big]\Big]. \end{aligned} \tag{11}$$

■

The proof of this Lemma and the other results of this Section can be found in [2]. The following theorem states how to solve recursively the GBR equation $\underline{E}_{X^t,Y^t}[I_{\{y^t\}}(g - \mu)] = 0$.

Theorem 2. Consider the same assumptions as in Lemma 1 and suppose that $\underline{E}_{X^t,Y^t}[I_{\{y^t\}}] > 0$ for any sequence of measurements y^t. Then, given $y^t = \{y_1, y_2, \ldots, y_t\}$ and a gamble $g : \mathcal{X}_t \to \mathbb{R}$, the separately coherent conditional lower prevision $\underline{E}_{X_t}[g|y^t]$ can be calculated by finding the unique value μ^* such that:

$$\begin{aligned} \mu^* &= \arg_\mu \left(\underline{E}_{X_0}[g_0] = 0\right), \text{ with} \\ g_{k-1}(x_{k-1}, \mu) &= \underline{E}_{X_k}\Big[g_k\Big(I_{\{g_k \geq 0\}}\underline{E}_{Y_k}[I_{\{y_k\}}|X_k] \\ &+ I_{\{g_k < 0\}}\overline{E}_{Y_k}[I_{\{y_k\}}|X_k]\Big)\Big|x_{k-1}\Big], \end{aligned} \tag{12}$$

for $k = 1, \ldots, t$, where $I_{\{g_k \geq 0\}}$ is the indicator of the set $\{x_k : g_k(x_k, \mu) \geq 0\}$, $I_{\{g_k < 0\}}$ its complement and $g_t(x_t, \mu) = g(x_t) - \mu$. ■

Notice that, the assumption $\underline{E}_{X^t,Y^t}[I_{\{y^t\}}] > 0$ can be met, since we have assumed the measurements to be discrete [2]. This is the *general solution* of the filtering problem in the case prior information on the state, state transition and measurement process are modelled through CLPs. The following corollary states that the general solution (12) comprises the classical Bayesian state estimation as a particular case, i.e., when CLPs reduce to standard expectations.

Corollary 1. Consider the same assumptions as in Theorem 2 and suppose that $\underline{E}_{X_0}[\cdot] = \overline{E}_{X_0}[\cdot] = E_{X_0}[\cdot]$, $\underline{E}_{X_k}[\cdot|X_{k-1}] = \overline{E}_{X_k}[\cdot|X_{k-1}] = E_{X_k}[\cdot|X_{k-1}]$ and $\underline{E}_{Y_k}[\cdot|X_k] = \overline{E}_{Y_k}[\cdot|X_k] = E_{Y_k}[\cdot|X_k]$. Then, $\underline{E}_{X_t}[g|y^t] = \overline{E}_{X_t}[g|y^t] = E_{X_t}[g|y^t]$, where:

$$E_{X_t}[g|y^t] = \frac{E_{X_{t-1}}\Big[E_{X_t}\Big[gE_{Y_t}[I_{\{y_t\}}|X_t]\Big|X_{t-1}\Big]\Big|y^{t-1}\Big]}{E_{X_{t-1}}\Big[E_{X_t}\Big[E_{Y_t}[I_{\{y_t\}}|X_t]\Big|X_{t-1}\Big]\Big|y^{t-1}\Big]}. \tag{13}$$

■

Furthermore, assuming some regularity conditions [6, Sec.6.10.4] for the existence of density functions, from (13) we can obtain Bayes' rule for conditional PDFs, i.e., $E_{X_t}[g|y^t]$ is equal to:

$$\frac{\int\limits_{x_{t-1}}\int\limits_{x_t} g(x_t)\;\; p(x_t|x_{t-1})p(x_{t-1}|y^{t-1})p(y_t|x_t)dx_t dx_{t-1}}{\int\limits_{x_{t-1}}\int\limits_{x_t} p(x_t|x_{t-1})p(x_{t-1}|y^{t-1})p(y_t|x_t)dx_t dx_{t-1}}. \tag{14}$$

Hence, $E_{X_t}[g|y^t]$ is a linear functional which is completely determined by the PDFs $p(x_t|x_{t-1})$, $p(x_{t-1}|y^{t-1})$ and $p(y_t|x_t)$.

In the classical bayesian estimation, once the posterior PDF $p(x_t|y^{t-1})$ has been computed, we can compute the posterior mean, by selecting $g(x_t) = x_t$ in (14), and its credible region (or confidence region). A $100(1-\alpha)$ credible region for a scalar random variable x is a region χ such that $E(I_{\{x\in\chi\}}|y^{t-1}) = Pr(x \in \chi|y^{t-1}) = 1-\alpha$, where $Pr(\cdot|y^{t-1})$ is the conditional posterior distribution. As discussed in Sec. 2.1, dealing with lower and upper expectations, we lead to the problem of decision making under imprecision [6]. For estimation, we follow the approach discussed in [2] which extends the Bayesian estimation approach to the CLP framework by calculating the lower $\underline{E}(x_t)$ and upper $\overline{E}(x_t)$ means and a CLP (robust) version of the credible region. In particular, the robust credible region is defined as the minimum volume region χ such that $\underline{E}(I_{\{x\in\chi\}}) > 1-\alpha$.

4 Multi-target tracking

The aim of this section is to apply CLPs and, in particular, the general solution of the filtering problem presented in Sec. (3) to the multi-target tracking problem. Target tracking from sensor observations is a difficult task, because of clutter and of the simultaneous presence of multiple targets whose probability to be detected is $P_d < 1$. At each sampling time, the sensor provides a set $Z_k = \{y_k^{(i)} : i = 1, 2, \ldots, m_k\}$ of measurements. These measurements are employed by the tracking system to perform two main tasks [1]: (i) filtering, i.e. the update of the target state using measurements and a model of the target motion; (ii) data association, i.e. the association of measurements to already established tracks.[5]

4.1 Filtering

In [2], the general solution presented in Section 3 has been specialised to a special class of coherent lower previsions, called linear Gaussian-vacuous mixtures (LGVM). In the sequel, we adopt the same model to perform the filtering task in the multi-target tracking problem. The basic idea in the LGVM filter is to model initial state and state transition by *linear Gaussian-vacuous mixtures*, i.e.,:

$$\underline{E}_{X_0}(g) = \epsilon_x \int\limits_{x_0} g(x_0)\,\mathcal{N}(x_0;\hat{x}_0,P_0)\,dx_0 + (1-\epsilon_x)\inf_{x_0} g(x_0), \tag{15}$$

$$\underline{E}_{X_t}(g|x_{t-1}) = \epsilon_w \int\limits_{x_t} g(x_t)\;\mathcal{N}(x_t;Ax_{t-1},Q)\;dx_t + (1-\epsilon_w)\inf_{x_t} g(x_t), \tag{16}$$

where the scalars ϵ_w and ϵ_x belong to $[0,1]$, This is equivalent to assume that the initial state and the time evolution of the state are described by $x_0 \sim \epsilon_x\mathcal{N}(\hat{x}_0,P_0) + (1-\epsilon_x)e_t$ and, respectively, $x_{t+1} = Ax_t + \epsilon_w w_t + (1-\epsilon_w)n_t$, where $w_t = \mathcal{N}(0,Q)$ and the noises n_t and e_t are assumed to have unknown distributions (not necessarily constant w.r.t time). Hence, n_t (and also e_t) accounts for possible unmodelled behaviours in the motion model. Since n_t can be arbitrary (any distribution), we can see (16) as a set of infinite motion models, i.e., all possible models obtained by perturbing the nominal model $Ax_t + \epsilon_w w_t$ with the unknown term $(1-\epsilon_w)n_t$. Note that the model which characterises both w_t^ϵ and x_0 is the so-called ϵ-*contamination* which has been defined in Sec. 2. The correspondence between this system and (15)–(16) has been discussed in [2]. Notice that, if $\epsilon_w = \epsilon_x = 1$, we are back to the model (7). As it has been shown in [2], when $\epsilon_w, \epsilon_x < 1$, LGVM is robust to unmodelled behaviours and, thus, is a perfect candidate to be used in multi-target tracking, where one of the main issues is to gain robustness w.r.t. to unmodelled manoeuvres. In [2], it is also shown that, if the measurement process is Gaussian and target moves according to the nominal model $x_{t+1} = Ax_t + w_t$, i.e., $n_t = 0$, then LGVM coincides with the Kalman filter based on the same model. Conversely, when target's behaviour differs from this nominal model, then LGVM is able to detect these situations and, thus, enlarging the credible region to include the true target's state.

4.2 Data association

For the measurement process, we adopt a different model from the one in [2]. In particular, we consider the possibility that, at each sampling time, the sensor returns more than one measurement (because of clutter and presence of other targets). Let $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_s\}$ be the set of all tracks associated to a moving targets. Assume instead of a single measurement, a vector of measurements $Z_k = [y_k^{(1)}, y_k^{(2)}, \ldots, y_k^{(m_k)}]^T$, i.e., $Z_k \in \mathcal{Z}_k \subseteq \mathbb{R}^{rm_k\times 1}$. Notice that: Z_k may be empty, since $P_d < 1$. The fact that Z_k may contain more than one measurement generates an ambiguity in the measurement process, which can be taken into account by means of CLPs. Assume that association measurements-track has already been addressed for any

[5] In this paper, we do not consider track initiation [1], i.e., we assume that tracks have already been initialized.

track τ_j from time 1 to time $k-1$. We denote with $Z_{\tau_j}^{k-1} = z_{\tau_j}^{k-1}$ the sequence of measurements already assigned to track τ_j. In the DA algorithms based on Bayesian probability, the likelihood that the measurement i at time k is generated by track τ is evaluated by computing the quantity $p^{\tau_j}(y_k^{(i)}, z_{\tau_j}^{k-1})$ (or the conditional $p^{\tau_j}(y_k^{(i)}|z_{\tau_j}^{k-1})$) which is equal to:

$$p^{\tau_j}(y_k^{(i)}, z_{\tau_j}^{k-1}) = E_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)], \quad (17)$$

where $p(y_k^{(i)}|x_k) = \mathcal{N}(y_k^{(i)}, Cx_k, R)$. Once all likelihoods $p^{\tau_j}(y_k^{(i)}, z_{\tau_j}^{k-1})$[6] have been computed for all measurements i at time k and tracks τ_j, DA can be performed by associating the pair measurement-track with highest likelihood, then the pair with second highest likelihood and so on. Association is thus based on the same decision process discussed in Sec. 2.1 in Eq. (5), i.e., by using the total order determined by $p^{\tau_j}(y_k^{(i)}, z_{\tau_j}^{k-1})$. This is, for instance, as the Global Nearest Neighbour works [1]. In the CLPs case, instead of $E_{X^k, Z_\tau^{k-1}}^{\tau_j}$, we deal with $\underline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau}$ and its conjugate upper prevision $\overline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau}$. Then, (17) becomes

$$\begin{aligned} \underline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)] \le p^{\tau_j}(y_k^{(i)}, z_\tau^{k-1}) \le \\ \overline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)]. \end{aligned} \quad (18)$$

Thus, instead of a single likelihood, for each measurement i we have an interval which is bounded by the lower and upper values given in (18). In this case, for decision making, we use the *interval dominance* criterion discussed in Sec. 2.1 and in Eq. (6). In particular, the association criterion used in this paper is described by the following algorithm. We refer to it as *Interval Dominance Data Association* (IDDA).

1. Initialize the set of tracks $\widetilde{\mathcal{T}} = \{\tau_1, \tau_2, \ldots, \tau_s\}$; the set of measurements at time k, $\widetilde{Z}_k = [y_k^{(1)}, y_k^{(2)}, \ldots, y_k^{(m_k)}]^T$[7]; the set of measurements associated to track τ_j, $\mathcal{A}^{\tau_j} = \{\emptyset\}$ for all $j = 1, \ldots, s$.
2. If $\widetilde{\mathcal{T}}$ and $\widetilde{Z}_k$ are not empty, select $\tau_j \in \widetilde{\mathcal{T}}$ and $y_k^{(i)}$ in $\widetilde{Z}_k$ such that $\underline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)]$ is maximum, else go to step 4.
3. For all $\tau_u \in \widetilde{\mathcal{T}}$ such that $\tau_u \neq \tau_j$, compute $\overline{E}_{X^k, Z_{\tau_u}^{k-1}}^{\tau_u}[I_{\{z_{\tau_u}^{k-1}\}} p(y_k^{(i)}|x_k)]$ and check if:
$$\begin{aligned} &\overline{E}_{X^k, Z_{\tau_u}^{k-1}}^{\tau_u}[I_{\{z_{\tau_u}^{k-1}\}} p(y_k^{(i)}|x_k)] \\ &< \underline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)]. \end{aligned} \quad (19)$$
If $\widetilde{\mathcal{T}} \neq \emptyset$ or if (19) is true for all τ_u, add measurement $y_k^{(i)}$ to $\mathcal{A}^{\tau_j}$, remove τ_j from $\widetilde{\mathcal{T}}$ and remove $y_k^{(i)}$ from $\widetilde{Z}_k$ and go back to 2.
4. Initialize again the set of tracks $\widetilde{\mathcal{T}}$ to $\{\tau_1, \tau_2, \ldots, \tau_s\}$;
5. If $\widetilde{\mathcal{T}}$ and $\widetilde{Z}_k \backslash \mathcal{A}^{\tau_j}$ are not empty, select $\tau_j \in \widetilde{\mathcal{T}}$ and $y_k^{(i)} \in \widetilde{Z}_k \backslash \mathcal{A}^{\tau_j}$ such that $\overline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)]$ is maximum, else go to step 8.
6. If $\mathcal{A}^{\tau_j}$ is empty then add $y_k^{(i)}$ to $\mathcal{A}^{\tau_j}$, else check if
$$\begin{aligned} &\overline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(i)}|x_k)] \\ &> \underline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}[I_{\{z_{\tau_j}^{k-1}\}} p(y_k^{(u)}|x_k)], \end{aligned} \quad (20)$$
for any other measurement $y_k^{(u)} \in \mathcal{A}^{\tau_j}$.
7. If (20) is met, add $y_k^{(u)}$ to $\mathcal{A}^{\tau_j}$ else go to step 8.
8. Remove τ_j from $\widetilde{\mathcal{T}}$ and go back to step 5.

To understand IDDA, first let us consider the case $\underline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j} = \overline{E}_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j} = E_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}$ for each track τ_j. In this case, we are back to GNN. Then inequality (19) is satisfied for all τ_u, while (20) is never satisfied, because $E_{X^k, Z_{\tau_j}^{k-1}}^{\tau_j}$ determines a total order. In the general case $\underline{E}_{X^k, Z_{\tau_e}^{k-1}}^{\tau_e} < \overline{E}_{X^k, Z_{\tau_e}^{k-1}}^{\tau_e}$ for some track τ_e, the algorithm first tries to solve DA globally by means of (19), i.e., comparing different tracks w.r.t. the same measurement. Then, for the unassigned measurements, steps 4–7 are performed. At the end of the procedure, if the set $Z_k^{\tau_j} := \mathcal{A}^{\tau_j} = \{y_k^{(j_1)}, \ldots, y_k^{(j_m)}\}$ includes just one measurement for all tracks τ_j, we are back to GNN. Conversely when $j_m > 1$, as discussed in Sec. 2.1, a way to robustly tackle this case is to consider all association hypotheses implied by the set $Z_k^{\tau_j}$. In particular, we address this ambiguity by considering the set of all possible likelihood-PDFs implied by $Z_k^{\tau_j}$, which is the same idea followed in MHT but interpreted in the CLPs framework.[8] This means that the measurement process is modelled by the following CLP:

$$\begin{aligned} \underline{E}_{Z_k^{\tau_j}}(h|x_k) \quad \propto \quad & \min_{i=0,j_1,\ldots,j_m} \sum_{z_k^\tau} h(z_k^{\tau_j}) \Big[(1-P_d) I_{\{i=0\}} \\ & + \quad I_{\{i\neq 0\}} \frac{P_d}{\lambda} \mathcal{N}(y_k^{(i)}, Cx_k, R)\Big], \end{aligned} \quad (21)$$

for each $h \in \mathcal{L}(\mathcal{Z}_k^\tau)$, $x_k \in \mathcal{X}_k$ and where λ is the clutter density. Notice that, we are considering also the missed

[6] Notice that, in the linear Gaussian case, $p^{\tau_j}(y_k^{(i)}, z_{\tau_j}^{k-1}) \propto \mathcal{N}(y_k^{(i)}; CA\hat{x}_{k-1}, S_k)$, where $\hat{x}_{k-1}$ is the estimate of the state based on the measurements $z_{\tau_j}^{k-1}$.

[7] Notice that, we are not performing gating, i.e., we consider all available measurements from the sensor.

[8] As in MHT, to reduce the computational load, we can implement gating and pruning strategies. Some ideas are also discussed in [2]. In this paper, since the simulation scenarios considered in Sec. 5 are not computationally intensive for IDDA, we have not implemented such strategies.

detection hypothesis, i.e., the term $1 - P_d$ associated to the index $i = 0$. Equation (21) defines a vacuous CLP over the set of indexes $j = 0, j_1, \ldots, j_m$ or, equivalently, over the set of hypotheses: (i) all entries of the measurement vector Z_k^τ are clutter generated and the target is not detected ($i = 0$); (ii) the measurement $y_k^{(i)}$ is target generated ($i \neq 0$). Notice also that, we have a sum instead of an integral in (21), because we have assumed the measurements to be discrete. The sum is over the space of the elements Z_k^τ, i.e., j_m variables. Once the minimum in (21) is computed, the result is just an expectation w.r.t. a uniform or Gaussian PDF:

$$\sum_{z_k^\tau} h(z_k^\tau)\Big[(1-P_d)I_{\{i'=0\}} + I_{\{i'\neq 0\}}\tfrac{P_d}{\lambda}\mathcal{N}(y_k^{(i')}, Cx_k, R)\Big] \tag{22}$$

where i' denotes the value $0, j_1, \ldots, j_m$ which obtains the minimum in (21). Similar expression holds for $\overline{E}_{Z_k^\tau}(h|x_k)$, but for a different i' (the one which obtains the maximum). Now that we have defined $\underline{E}_{X_0}$, $\underline{E}_{X_k}(\cdot|x_{k-1})$ in (15)–(16) and $\underline{E}_{Z_k^\tau}(\cdot|x_k)$ in (21) for each $k \geq 1$, we can derive $\underline{E}^\tau_{X_t}[g|Z^t_\tau = z^t_\tau]$ for all tracks τ and function of interest g, as discussed in Sec. 3. In the following, for decision making purposes and to evaluate the performance of the proposed multi-target tracker, we will use the 99% posterior robust credible region, i.e., the minimum volume region $\chi \subseteq \mathbb{R}^n$ such that $\underline{E}^\tau_{X_t}[I_\chi|z^t] > 0.99$. In [2], a specialised version of the results in Sec. 3 has been derived for linear Gaussian-vacuous mixture, by exploiting the properties of Gaussian PDFs to simplify the computations. Since (15)–(16) and (22) are indeed linear Gaussian-vacuous mixture, we have used the same derivations to solve (12). We refer to [2, Sec. 5] for the details.

5 Performance evaluation

In this section, the performance of the tracker obtained by combining the LGVM filter and IDDA algorithm is assessed by means of Monte Carlo simulations, concerning different scenarios. The simulated targets move in the xy Cartesian plane and are, therefore, characterized, at discrete time t, by the state vector $x_t = [p_x, v_x, p_y, v_y]^T$, where (p_x, p_y) provides the position and (v_x, v_y) the velocity in Cartesian coordinates at time t. The following motion model has been considered for the targets:[9]

$$\begin{cases} x_{t+1} &= Ax_t + w_t^\epsilon \\ y_t &= Cx_t + v_t \end{cases}$$
$$A = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{23}$$

[9]Notice that, if measurements were in polar coordinates, we could use linearisation and reduce again to the linear case, but with a time-varying covariance R.

where $T = 1$ is the sampling interval; $w_t^\epsilon = \epsilon_w w_t + (1-\epsilon_w)n_t$, $w_t \sim \mathcal{N}(0, Q)$, $x_0 = \hat{x}_0$ (i.e., $\epsilon_x = 1$), $\hat{x}_0 \sim N(0, P_0)$, $v_t \sim N(0, R)$, $P_0 = diag\,[p_0, p_0, p_0, p_0]$, $Q = diag\,[q, q, q, q]$, $R = diag\,[r, r]$ with $p_0 = 0.2$,$q = 0.1$, $r = 0.1$, $T = 1$, $\epsilon_w = 0.9999$. It has also been assumed that the two components of the state are constrained to lie in $[-150, 150]$ and, respectively, $[-30, 30]$. The performance of the multi-target tracker will be investigated by considering different distributions for the contaminating term n_t and, thus, different manoeuvres. If not otherwise stated, we consider a scenario with 2 targets, trajectories' length of 40 time instants, clutter presence (5 clutter measurements at each time instant), $\lambda = 10^{-6}$ and 100 Monte Carlo (MC) runs. Notice that, MC runs have been performed only w.r.t the measurement's noise realisations (the trajectory is fixed). In this way, we can force the two targets to cross their trajectories at the same time. To evaluate the performance of the proposed algorithm, the following metrics will be considered.

AE (*Association Error*): a binary variable taking value 0 if the measurement associated to track i at time t is the right one and value 1 otherwise

TC (*Track Continuity*): a real variable in $[0, 1]$ that measures the percentage of trajectory in which targets are continuously represented by the same track (i.e., target position included in the robust 99% credible region of the associated track).

All these metrics are averaged over the number of targets, the trajectory's duration and the number of MC trials.

Scenario 1: Let us first consider the multitarget case-study depicted in Fig. 1. The initial states of the two targets are $[0, 1, 24, -1]$ and, respectively, $[0, 1, -24, 1]$. Targets move according to the motion model (23) with $n_k = [0, 0, 0, 0]^T$ for each $15 \neq k \in \{1, \ldots, 40\}$, while n_{15} is equal to $[0, 0, -4, 0]^T$ for target 1 (red line) and to $[0, 0, 4, 0]^T$ for target 2 (blue line). This means that the trajectories of the two targets undergo a jump of 4 units along the y-axis at the time instant $k = 15$. This can be interpreted as an manoeuvre. Fig. 1 also shows the true position of the first target at each instant (denoted by a red asterisk mark) and the estimated 99% robust credible regions (red ellipse) (for the case $P_d = 1$ and no-clutter). We can see that, at the instant $k = 15$, the tracker correctly detects the manoeuvre and it is able to enlarge the credible region in order to include the true state. The tracker, because of LGVM filter, is thus able to distinguish situations where tracking is easy, i.e., targets are not manoeuvring (small credible regions) to situations where tracking is difficult, i.e., targets are manoeuvring (large credible regions). In this case, at the trajectory-crossing, IDDA gives the same answer of GNN, since there is not uncertainty (i.e., targets are not manoeuvring). The simulation results are shown in table 1 in terms of AE and TC for different values of P_d and clutter presence. We can see that the algorithm

Figure 1: Case 1: 99% robust cred. regions relative to a single MC run for the 1st target, $P_d = 1$ and no-clutter.

	AE	**TC**
$P_d = 0.8$	0.06	0.94
$P_d = 0.9$	0.03	1
$P_d = 1$	0.02	1

Table 1: Simulations results for scenario 1

performs well also at the decreasing of the detection probability.

Scenario 2: let us now consider the multi-target case-study depicted in Fig. 2. The initial states of the two targets are $[0, 1, 16, -1]$ and, respectively, $[0, 1, -24, 1]$. Targets move according to the motion model (23) with $n_k = [0, 0, 0, 0]^T$ for each $20 \neq k \in \{1, \ldots, 40\}$, while n_{20} is equal to $[0, 0, -4, 0]^T$ for target 1 (red line) and to $[0, 0, +4, 0]^T$ for target 2 (blue line). This means that the targets execute a manoeuvre at the crossing. Fig. 2 also shows the estimated 99% robust credible region. We can see that, from time 20 to 24, the credible regions are very large. In this case, the IDDA algorithm response is different from that would be the one from GNN. Because of the manoeuvre, we cannot in fact decide which measurement is originated from which target. Hence, the tracker propagates all possible association hypotheses until a decision can be reached (as the MHT would do in this case). However, conversely to MHT, the proposed tracker do not propagate all possible hypotheses separately, but jointly by means of their lower and upper envelope (which is a CLP). Thus, from time 20 to 24, the credible regions shown in Fig. 2 include both targets's states, because of the uncertainty in DA. The simulation results in terms of AE and TC are shown in table 2.

	AE	**TC**
$P_d = 0.8$	0.15	0.84
$P_d = 0.9$	0.09	0.89
$P_d = 1$	0.04	0.93

Table 2: Simulations results for scenario 2

Figure 2: Case 2: 99% robust cred. regions relative to a single MC run for the 1st target, $P_d = 1$ and no-clutter.

6 Conclusions

In this paper, a new multi-target tracker has been proposed based on closed-convex sets of probability distributions used to characterize uncertainty in the prior, target's motion and in the origin of the measurements (target or clutter). We have also shown, in practical cases, that this new multi-target tracker is able to robustly deal with difficult tracking scenarios (manoeuvring and crossing targets). As future prospects, we intend to investigate the performance of the tracker in more difficult scenarios (more targets, more clutter etc.). We also plan to investigate track initiation, which has not be addressed in this paper.

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