# MINIMUM POWER SYMMETRIC CONNECTIVITY PROBLEM IN WIRELESS NETWORKS: A NEW APPROACH

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**Abstract** We consider the problem of assigning transmission powers to the nodes of a wireless network in such a way that all the nodes of the network are connected by bidirectional links and the total power consumption is minimized.

A new exact algorithm, based on a new integer programming model, is described in this paper together with a new preprocessing technique.

**Keywords:** Wireless networks, minimum power topology, exact algorithms.

## 1. Introduction

Ad-hoc wireless networks have been significantly studied in the past few years due to their potential applications in battlefield, emergency disasters relief, and other application scenarios (see, for example, Singh et al., 1999, Ramanathan and Rosales-Hain, 2000, Wieselthier et al., 2000, Wan et al., 2001 and Lloyd et al., 2002). Unlike wired networks of cellular networks, no wired backbone infrastructure is installed in ad-hoc wireless networks. A communication session is achieved either through single-hop transmission if the recipient is within the transmission range of the source node, or by relaying through intermediate nodes otherwise.

We consider the problem of minimum transmit power bidirectional topology in multi-hop wireless networks where individual nodes are typically equipped with limited capacity batteries and therefore have a restricted lifetime. Topology control is one of the most fundamental and critical issues in multi-hop wireless networks which directly affect the

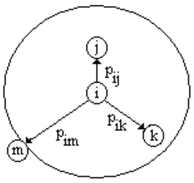


*Figure 1.* Wireless communication model.

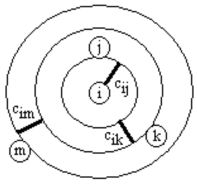


*Figure 2.* Costs for the mathematical formulation $IP$.

network performance. In wireless networks, topology control essentially involves choosing the right set of transmitter power to maintain adequate network connectivity. In energy-constrained networks, where replacement or periodic maintenance of node batteries is not feasible, the issue is very critical since it directly impacts the network lifetime.

In Ramanathan and Rosales-Hain, 2000 the problem of controlling topology using transmission power control in wireless networks is firstly approached in terms of optimization. It is showed that a network topology which minimizes the maximum transmitter power allocated to any node can be constructed in polynomial time. This is a critical criterion in battlefield applications since using higher transmitter power increases the probability of detection by enemy radar. In this paper, we focus on the minimum power topology problem in wireless networks with omnidirectional antennae. It has been shown in Clementi et al., 1999 that this problem is NP-complete. Related work in the area of minimum power topology construction include Wattenhofer et al., 2001 and Huang et al., 2002, which propose distributed algorithms.

Unlike in wired networks, where a transmission from $i$ to $m$ generally reaches only node $m$, in wireless networks with omnidirectional antennae it is possible to reach several nodes with a single transmission (this is the so-called *wireless multi-cast advantage*, see Wieselthier et al., 2000). In the example of Figure 1 nodes $j$ and $k$ receive the signal originated from node $i$ and directed to node $m$ because $j$ and $k$ are closer to $i$ than $m$, i.e. they are within the transmission range of a communication from $i$ to $m$. This property is used to minimize the total transmission power required to connect all the nodes of the network. For a given set of nodes, the *Minimum Power symmetric Connectivity* ($MPC$) *problem* is to assign transmission powers to the nodes of the network in such a way that all the nodes are connected by bidirectional links and the total power consumption over the network is minimized. Having bidirectional links simplifies one-hop transmission protocols by allowing acknowledgement messages to be sent back for every packet (see Althaus et al., 2003). It is assumed that no power expenditure is involved in reception/processing activities, that a complete knowledge of pairwise distances between nodes is available, and that there is no mobility.

## 2. Problem description

In order to formalize the problem, a model for signal propagation has to be selected. We adopt the model presented in Rappaport, 1996. Signal power falls as $\frac{1}{d^\kappa}$, where $d$ is the distance from the transmitter to the receiver and $\kappa$ is an environment-dependent coefficient, typically between 2 and 4 (we will set $\kappa = 4$). Under this model, and adopting the usual convention (see, for example, Althaus et al., 2003 and Montemanni et al., 2004) that every node has the same transmission efficiency and the same detection sensitivity threshold, the power requirement for supporting a link from node $i$ to node $j$, separated by a distance $d_{ij}$, is then given by

$$p_{ij} = (d_{ij})^\kappa \tag{1}$$

It is important to notice that the results presented in this paper are still valid in case more complex signal propagation models more complex are taken into account.

We assume that there is no constraint on maximum transmission powers of nodes. However, the algorithm we discuss in this paper can be extended straightforwardly to the case when this assumption does not hold. If, for example, node $i$ cannot reach node $j$ even when it is transmitting to its maximum power (i.e. $d_{ij}^\kappa >$ maximum power of node $i$), then $p_{ij}$ can be redefined as $+\infty$.

$MPC$ can be formally described as follows:
Given the set $V$ of the nodes of the network, a *range assignment* is a function $r : V \to \mathcal{R}^+$. A *bidirectional link* between nodes $i$ and $j$ is said to be established under the range assignment $r$ if $r(i) \geq p_{ij}$ and $r(j) \geq p_{ij}$. Let now $B(r)$ denote the set of all bidirectional links established under the range assignment $r$. $MPC$ is the problem of finding a range assignment $r$ minimizing $\sum_{i \in V} r(i)$, subject to the constraint that the graph $(V, B(r))$ is connected.

As suggested in Althaus et al., 2003, a graph theoretical description of $MPC$ can be given as follows:
Let $G = (V, E, p)$ be an edge-weighted graph, where $V$ is the set of vertices corresponding to the set of nodes of the network and $E$ is the set of edges containing all the possible (unsorted) pairs $\{i, j\}$, with $i, j \in V$, $i \neq j$. A cost $p_{ij}$ is associated with each edge $\{i, j\}$. It corresponds to the power requirement defined by equation (1).

For a node $i$ and a spanning tree $T$ of $G$ (see, for example, Kruskal, 1956), let $\{i, i_T\}$ be the maximum cost edge incident to $i$ in $T$, i.e. $\{i, i_T\} \in T$ and $p_{ii_T} \geq p_{ij}$ $\forall \{i,j\} \in T$. The *power cost* of a spanning tree $T$ is then $c(T) = \sum_{i \in V} p_{ii_T}$. Since any connected graph contains a spanning tree, and a broadcast tree must be connected, $MPC$ can be described as the problem of finding the spanning tree $T$ with minimum power cost $c(T)$. This observation is at the basis of the integer programming formulation which will be presented in Section 3.

## 3. An integer programming formulation

A weighted, directed graph $G' = (V, A, p)$ is derived from $G$ by defining $A = \{(i,j),(j,i) | \{i,j\} \in E\} \cup \{(i,i) | i \in V\}$, i.e. for each edge in $E$ there are the respective two (oriented) arcs in $A$, and a dummy arc $(i,i)$ with $p_{ii} = 0$ is inserted for each $i \in V$. $p_{ij}$ is defined by equation (1) when $i \neq j$. In order to describe the new integer programming formulation for $MPC$, we also need the following definition.

Given $(i,j) \in A$, we define the *ancestor* of $(i,j)$ as

$$a^i_j = \begin{cases} i & \text{if } p_{ij} = \min_{\{i,k\} \in E}\{p_{ik}\} \\ arg\max_{k \in V}\{p_{ik} | p_{ik} < p_{ij}\} & \text{otherwise} \end{cases} \tag{2}$$

According to this definition, $(i, a^i_j)$ is the arc originated in node $i$ with the highest cost such that $p_{ia^i_j} < p_{ij}$. In case an *ancestor* does not exist for arc $(i,j)$, vertex $i$ is returned, i.e. the dummy arc $(i,i)$ is addressed.

In formulation $IP$ a spanning tree (eventually augmented) is defined by $z$ variables: $z_{ij} = 1$ if edge $\{i,j\}$ is on the spanning tree, $z_{ij} = 0$ otherwise. Variable $y_{ij}$ is 1 when node $i$ has a transmission power which allows it to reach node $j$, $y_{ij} = 0$ otherwise.

$$(IP) \quad \text{Min} \sum_{(i,j) \in A} c_{ij} y_{ij} \tag{3}$$

$$\text{s.t. } y_{ij} \leq y_{ia^i_j} \qquad \forall (i,j) \in A, a^i_j \neq i \tag{4}$$

$$z_{ij} \leq y_{ij} \qquad \forall \{i,j\} \in E \tag{5}$$

$$z_{ij} \leq y_{ji} \qquad \forall \{i,j\} \in E \tag{6}$$

$$\sum_{i \in S, j \in V \setminus S, \{i,j\} \in E} z_{ij} \geq 1 \qquad \forall S \subset V \tag{7}$$

$$z_{ij} \in \{0,1\} \qquad \forall \{i,j\} \in E \tag{8}$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{9}$$

In formulation $IP$ an incremental mechanism is established over $y$ variables (i.e. transmission powers). The costs associated with $y$ variables in the objective function (3) are given by the following formula:

$$c_{ij} = p_{ij} - p_{ia^i_j} \quad \forall (i,j) \in A \tag{10}$$

$c_{ij}$ is equal to the power required to establish a transmission from node $i$ to node $j$ ($p_{ij}$) minus the power required by node $i$ to reach node $a^i_j$ ($p_{ia^i_j}$). In Figure 2 a pictorial representation of the costs arising from the example of Figure 1 is given.

Constraints (4) realize the incremental mechanism by forcing the variable associated with arc $(i, a^i_j)$ to assume value 1 when the variable associated with arc $(i,j)$ has value 1, i.e. the arcs originated in the same node are activated in increasing order of $p$. Inequalities (5) and (6) connect the spanning tree variables $z$ to transmission power variables $y$. Basically, given edge $\{i,j\} \in E$, $z_{ij}$ can assume value 1 if and only if both $y_{ij}$ and $y_{ji}$ have value 1. Equations (7) state that all the vertices have to be mutually connected in the subgraph induced by $z$ variables, i.e. the (eventually augmented) spanning tree. Constraints (8) and (9) define variable domains.

## 3.1 Valid inequalities

A set of valid inequalities is proposed in Montemanni and Gambardella, 2003 for a formulation described in the same paper. Most of these inequalities can be easily adapted to formulation $IP$, and the remainder of this section is devoted to their description in terms of formulation $IP$.

In order to describe these valid inequalities, we will refer to the subgraph of $G'$ defined by the $y$ variables with value 1 as $G_y$. Formally, $G_y = (V, A_y)$, where $A_y = \{(i,j) \in A | y_{ij} = 1 \text{ in the current solution of } IP\}$.

**Connectivity inequalities:** since graph $G_y$ must be connected by definition, each node $i$ must be able to communicate with at least another node. Its transmission power must then be sufficient to reach at least the node $j$ which is closest to it. This can be expressed through the following set of inequalities:

$$y_{ij} = 1 \quad \forall (i,j) \in A \text{ s.t. } a^i_j = i \tag{11}$$

**Bidirectional inequalities 1:** for each arc $(i,j) \in A$, if $y_{ij} = 0$ and $y_{ia^i_j} = 1$ then the transmission power of node $i$ is set to reach node $a^i_j$ and nothing more. The only reason for node $i$ to reach node $a^i_j$ and nothing more is the existence of a bidirectional link on edge $\{i, a^i_j\}$ in

$G_y$. Consequently $y_{a^i_j i}$ must be equal to 1. This is what the following set of constraints states.

$$y_{a^i_j i} \geq y_{ia^i_j} - y_{ij} \quad \forall (i,j) \in A \text{ s.t. } a^i_j \neq i \tag{12}$$

Notice that if $y_{ij} = 1$ then $y_{ia^i_j} = 1$ because of inequalities (4) and consequently in this case the constraint does not give any new contribution. If $y_{ij} = 0$ and $y_{ia^i_j} = 0$ then again the constraint does not give any new contribution.

**Bidirectional inequalities 2:** consider arc $(i,j) \in A$, where $j$ is the farthest node from $i$ (i.e. $\nexists (i,k) \in A, a^i_k = j$) and suppose $y_{ij} = 1$. The only reason for node $i$ to reach node $j$ is the existence of a bidirectional link on edge $\{i,j\}$ in $G_y$. Consequently $y_{ji}$ must be equal to 1, as stated by the following set of constraints.

$$y_{ji} \geq y_{ij} \;\; \forall (i,j) \in A \text{ s.t. } \nexists (i,k) \in A, a^i_k = j \tag{13}$$

Notice that if $y_{ij} = 0$ the constraint does not give any contribution to formulation $IP$.

**Tree inequality:** in order to be strongly connected, the directed graph $G_y$ must have at least $2(|V|-1)$ arcs, as stated by the following constraint.

$$\sum_{(i,j) \in A} y_{ij} \geq 2(|V|-1) \tag{14}$$

**Reachability inequalities 1:** in order to define this set of valid inequalities, we need the following definitions.

$G_a = (V, A_a)$ is the subgraph of the complete graph $G'$ such that $A_a = \{(i,j) |\ a^i_j = i\}$. Notice that $|A_a| = |V|$ by definition.

$\mathcal{R}_i = \{j \in V |\ j$ can be reached from $i$ in $G_a\}$.

The inequalities are based on the consideration that, since graph $G_y$ must be strongly connected, it must be possible to reach every node $j$ starting from each node $i$. This implies that at least one arc must exist between the nodes which is possible to reach from $i$ in $G_a$ (i.e. $\mathcal{R}_i$) and the other nodes of the graph (i.e. $V \backslash \mathcal{R}_i$). The following set of inequalities arises:

$$\sum_{(k,l) \in A,\ k \in \mathcal{R}_i, l \in V \backslash \mathcal{R}_i} y_{kl} \geq 1 \quad \forall i \in V \tag{15}$$

**Reachability inequalities 2:** in order to define this set of valid inequalities, we need the following definition.

$\mathcal{Q}_i = \{j \in V |\ i$ can be reached from $j$ in $G_a\}$.

These inequalities are based on the idea that, since graph $G_y$ must be strongly connected, it must be possible to reach every node $i$ from every other node $j$ of the graph. This means that at least one arc must exist between the nodes which cannot reach $i$ in $G_a$ (i.e. $V\backslash\mathcal{Q}_i$) and the other nodes of the graph (i.e. $\mathcal{Q}_i$). The following set of constraints arises:

$$\sum_{(l,k)\in A, l\in\mathcal{Q}_i,\ k\in V\backslash\mathcal{Q}_i} y_{lk} \geq 1 \quad \forall i \in V \tag{16}$$

In the remainder of this paper we will refer to formulation $IP$ reinforced with inequalities (11)-(16) as $IP_R$. We will use this last, reinforced formulation because the extra inequalities strictly constrain $y$ variables to assume quasi-feasible values (in terms of $IP^{PMB}$) only and this leads to much shorter solving times (see Montemanni and Gambardella, 2003).

## 4. Preprocessing procedure

The theoretical result described in this section is used to reduce the number of edges of a problem (and consequently the number of variables of formulation $IP$).

Given a problem, we suppose we have an heuristic solution, *heu*, with cost $cost(heu)$ for it. Given a node $i$, all its transmission power levels that, if implemented, would induce a cost higher than $cost(heu)$ can be ignored. More formally:

**Theorem 1.** *If the following inequality holds*

$$2p_{ij} + \sum_{k\in V\backslash\{\{i\}\cup\{j\}\},\ a_l^k=k} p_{kl} > cost(heu) \tag{17}$$

*then edge* $\{i,j\}$ *can be deleted from* $E$.

*Proof.* If $p_{ij}$ is the power of node $i$ in a solution, this means that the power of node $j$ must be greater than or equal to $p_{ji}(=p_{ij})$, i.e. arc $(j,i)$ must be in the solution, because otherwise there would be no reason for node $i$ to reach node $j$. The sum in the left hand side of the inequality represents a lower bound for the power required by nodes different from $i$ and $j$ to maintain the network connected. The left hand side of inequality (17) represents then a lower bound for the total power required in case node $i$ transmits to a power which allows it to reach node $j$ and nothing farther. For this reason, if inequality (17) holds, edge $\{i,j\}$ can be deleted from $E$. □

It is important to notice that once edge $\{i,j\}$ is deleted from $E$, the value of the ancestor of arc $(i,k)$ ($(j,l)$) with $a_k^i = j$ ($a_l^j = i$) has to be updated to $a_j^i$ ($a_i^j$).

## 5. The iterative exact algorithm *IEX*

In this section we describe an algorithm which solves to optimality formulation $IP$ (i.e. the minimum power symmetric connectivity problem).

It is very difficult to deal with constraints (7) of formulation $IP_R$ in case of large problems. For this reason some techniques which leave some of them out have to be considered. We present an iterative algorithm ($IEX$) which in the beginning does not consider constraints (7) at all, and then adds them step by step only in case they are violated.

In order to speed up the approach, the following inequality should also be added to the initial integer problem $IP_R$:

$$\sum_{\{i,j\}\in E} z_{ij} \geq |V| - 1 \tag{18}$$

Inequality (18) forces the number of active $z$ variables to be at least $|V| - 1$ - this condition is necessary in order to have a spanning tree - already at the very first iterations of the algorithm.

The integer program defined as $IP_R$ without constraints (7) but with inequality (18), is solved and the values of the $z$ variables in the solution are examined. If the edges corresponding to $z$ variables with value 1 form a spanning tree then the problem has been solved to optimality, otherwise constraints (19), described below, are added to the integer program and the process is repeated.

At the end of each iteration, the last available solution is examined and, if edges corresponding to $z$ variables with value 1 generate a set $\mathcal{CC}$ of connected components with $|\mathcal{CC}| > 1$, then the following inequalities are added to the formulation:

$$\sum_{i\in C, j\in V\setminus C,\ \{i,j\}\in E} z_{ij} \geq 1 \quad \forall C \in \mathcal{CC} \tag{19}$$

Inequalities (19) force $z$ variables with value 1 to connect the (elsewhere disjoint) connected components in $\mathcal{CC}$ to each other.

## 6. Computational results

In this section we present some experiments aiming to evaluate the performance of the preprocessing technique described in Section 4 and of the exact algorithm presented in Section 5.

Tests have been carried out on problems randomly generated as described in Althaus et al., 2003. For each problem of size $|V|$ generated, $|V|$ points - they are the nodes of the network - have been chosen uniformly at random from a grid of size 10000 $\times$ 10000.

The preprocessing technique and the $IEX$ algorithm have been implemented in ANSI C, and the callable library of ILOG CPLEX 6.0 (see *http://www.cplex.com*) has been used to solve the integer programs encountered during the execution algorithm $IEX$. Tests have been carried out on a SUNW Ultra-30 machine.

In this paper consider networks with up to 50 nodes, but it is important to notice that in case the algorithm is used within a distributed/dynamic environment, the typical local vision of a node can be estimated in a few tens of nodes, that may be reflected into a much larger global network.

## 6.1 Preprocessing procedure

In order to apply the preprocessing procedure described in Section 4, a heuristic solution to the problem must be available. For this purpose we use one of the simplest algorithms available, MST, which works by calculating the *Minimum Spanning Tree* $T$ (see Prim, 1957) on the weighted graph with costs defined by equation (1), and by assigning the power of each transmitter $i$ to $p_{ii_T}$, as described near the end of Section 2. More complex algorithms, which guarantee better performance, have been proposed (see, for example, Althaus et al., 2003). It is worth to observe that if these algorithms have had been adopted, also the preprocessing technique would have produced better results than those reported in the remainder of this section.

In Table 1 we present, for different values of $|V|$, the average percentage of arcs deleted by the preprocessing procedure over fifty runs.

Table 1 suggests that the preprocessing technique we propose dramatically simplifies problems. It is also interesting to observe that the percentage of arcs deleted considerably increases when the number of nodes ($|V|$) increases. This means that, when dimensions increase, the extra complexity induced by extra nodes is partially mitigated by the increased efficiency of the preprocessing technique. This should help to contain the complexity explosion faced when the number of nodes goes up.

## 6.2 $IEX$ algorithm

In Table 2 we present the average computation times required by different exact algorithms to solve to optimality problems for different values of $|V|$. Fifty instances have been considered for each value of $|V|$.

The results in the second column of Table 2 are those presented in Althaus et al., 2003 (obtained on an AMD Duron 600MHz PC) multiplied by a factor of 3.2 (as suggested in Dongarra, 2003). This makes

*Table 1.* Preprocessing technique. Average performance.

| $\|V\|$ | Arcs deleted (%) |
|---|---|
| 10 | 57.556 |
| 15 | 63.781 |
| 20 | 66.526 |
| 25 | 70.393 |
| 30 | 72.464 |
| 35 | 74.647 |
| 40 | 76.106 |
| 45 | 77.568 |
| 50 | 78.688 |

*Table 2.* Exact algorithms. Average computation times (sec).

| $\|V\|$ | Althaus et al., 2003 | Montemanni and Gambardella, 2003 | $IEX$ |
|---|---|---|---|
| 10 | 2.144 | 0.192 | 0.052 |
| 15 | 18.176 | 0.736 | 0.196 |
| 20 | 71.04 | 8.576 | 0.601 |
| 25 | 188.48 | 33.152 | 2.181 |
| 30 | 643.2 | 221.408 | 13.481 |
| 35 | 2278.4 | 1246.304 | 28.172 |
| 40 | 15120 | 9886.08 | 79.544 |

them comparable with the other results of the table. In the third column the results presented in Montemanni and Gambardella, 2003 are summarized. Table 2 shows that algorithm $IEX$ outperforms, in terms of time required to retrieve optimal solutions, the other exact methods. In particular it is important to observe that the gap between the computational times of this algorithm and those of the other methods tends to increase when the number of nodes considered increases.

## 7. Conclusion

In this paper we have considered the problem of assigning transmission powers to the nodes of a wireless network in such a way that all the nodes are connected by bidirectional links and the total power consumption is minimized.

We have presented a new integer programming formulation for the problem and a new algorithm, based on this formulation, which retrieves optimal solutions according to the model considered. A preprocessing technique has been also proposed.

We are currently researching on a framework where the algorithm we propose is used locally at each node of a distributed/dynamic network. The objective will be to evaluate the behavior of our novel approach in this context.

## Acknowledgments

The work was partially supported by the FET unit of the European Commission through project BISON (IST-2001-38923).

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