Why should we use particle filtering in FM band passive radars?

A. Benavoli, A. Di Lallo

SELEX Sistemi Integrati

Via Tiburtina km 12.4, 00131 Rome, Italy

abenavoli@selex-si.com

adilallo@selex-si.com

***Abstract*— This paper shows that innovative tracking algorithms improve the state estimation performance of a Passive Covert Radar (PCR) exploiting a single non co-operative frequency modulated (FM) commercial radio station as its transmitter of opportunity. In particular, since target tracks in the Cartesian domain are obtained via particle filtering (PF), this paper aims at assessing its performance with respect to conventional tracking techniques such as EKF (Extended Kalman Filter) and at showing the improvement it yields.**

I. INTRODUCTION

This paper refers to the signal and data processing sections of a FM radio based bi-static radar described in . The main technical advance that has been achieved with respect to previous experimental works (e.g., [2]) is the improvement of target state estimation: target direction of arrival is now effectively integrated in the tracking algorithm and the application of particle filtering to confirmed tracks properly transforms target bi-static range, Doppler and bearing into Cartesian position and velocity. This paper presents the rationale for using particle filtering instead of more conventional tracking techniques. Simulation results do in fact validate and quantify the improvement PF yields in terms of position and velocity estimate accuracy.

This paper is organized as follows: section II gives a brief recall of the signal processing algorithms. Section III illustrates the data processing design. Section IV presents and compares the performance of the tracking algorithms. Conclusions are given in section V.

II. SIGNAL PROCESSING

The signal processing block diagram is depicted in Fig. 1 [2]: "Right" and "Left" correspond to the digital signals generated by the two channels of the interferometric surveillance system, whereas "Reference" is the binary data stream produced by the reference channel.

The main functions of the developed signal processor are:

- adaptive direct signal interference cancellation;
- Doppler-sensitive cross-correlation search for target echoes, using the reference signal as the optimal matched filter that provides the necessary processing gain;
- adaptive thresholding used to automatically detect targets on the two range-Doppler maps (associated to left and right channels) maintaining a constant false alarm probability via a custom bi-dimensional Cell-Average CFAR (Constant False Alarm Rate);
- "2/2" logic to extract detections common to right and left channels;
- bearing estimation based on the use of a simple phase interferometry. The difference between the arguments of the two range-Doppler surfaces is unambiguously related to the target direction of arrival in the azimuth sector the radar surveys, which is about ± 30°.

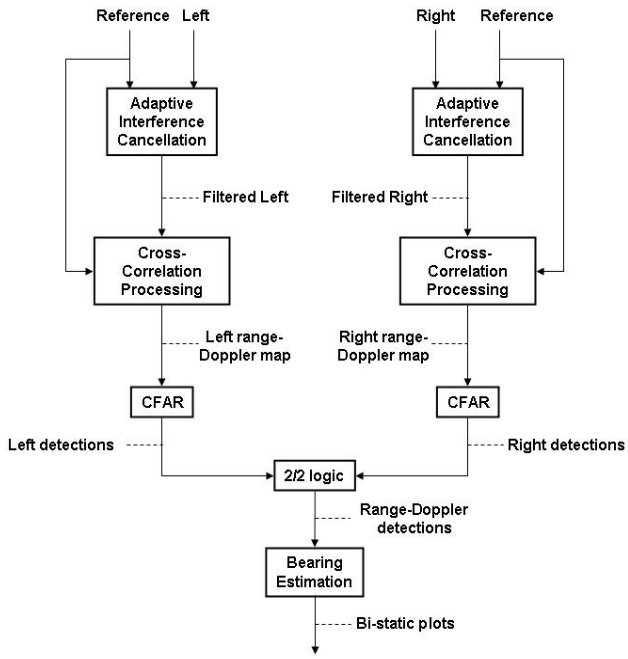

Fig. 1 Signal processing block diagram

III. DATA PROCESSING

The block diagram in Fig. 2 shows the algorithms that have been designed and developed to track the targets the bi-static passive radar detects.

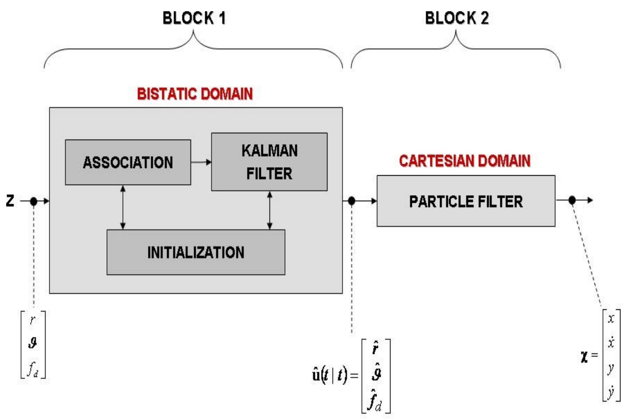

Fig. 2 Data processing block diagram

The working principle of the implemented data processor is the following. Plots relative to slowly manoeuvring targets, clutter and multipath are collected in the measurement vector $\mathbf{Z}$ associated to the current scan (the radar system needs about 2.5 seconds to fully process the 1.3 second's worth of received signals, which corresponds to a sort of conventional radars scan period). Each plot is a set of three measurements: bi-static range r, azimuth ϑ and bi-static Doppler frequency f_d. The measurement vector $\mathbf{Z}$ feeds the tracking algorithm, which is formed by two cascading blocks. The former implements track initialization, plot to track association and recursive filtering in the bi-static domain; in particular, target motion model linearity permits using a simple Kalman filter, as suggested in [2]. The need for Cartesian tracks to be compared with simulated truth data has suggested the Kalman-filtered measurement vector should be followed by a particle filter . Therefore the second block of the tracking architecture that has been implemented consists of a particle filter that applies to the output (i.e., confirmed tracks) of the previous block to estimate target tracks in the Cartesian domain.

Block 2 of Fig. 2 receives the filtered measurement vector $\hat{\mathbf{u}}(t \mid t)$ from Block 1 and estimates target motion in Cartesian coordinates $\boldsymbol{\chi} = [x, \dot{x}, y, \dot{y}]$, according to Equation (1):

$$\hat{\mathbf{u}}(t \mid t) = h(\boldsymbol{\chi}) + \mathbf{v} \tag{1}$$

where

$$h(\boldsymbol{\chi}) = \begin{bmatrix} r_{Rx} + r_{Tx} - d \\ \measuredangle(x + i\,y) \\ -\frac{1}{\lambda}\left(\frac{x\,\dot{x} - y\,\dot{y}}{\sqrt{x^2 + y^2}} + \frac{(x - x_{Tx})\dot{x} - (y - y_{Tx})\dot{y}}{\sqrt{(x - x_{Tx})^2 + (y - y_{Tx})^2}} \right) \end{bmatrix} \tag{2}$$

The elements of the three-dimensional $h(\boldsymbol{\chi})$ in Equation (2) do respectively represent: bi-static range, bearing, bi-static Doppler frequency. They depend on:

- distance d receiver-transmitter;
- distance r_{Rx} target-receiver, and distance r_{Tx} target-transmitter;
- transmitter position $[x_{Tx}, y_{Tx}]$ (the receiver lies at the centre of the reference system).

The target is assumed to move with uniform speed along a straight line, in accordance with Equations (3) and (4):

$$\begin{bmatrix} x(k+1) \\ \dot{x}(k+1) \\ y(k+1) \\ \dot{y}(k+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ y(k) \\ \dot{y}(k) \end{bmatrix} + \mathbf{w} \tag{3}$$

$$\mathbf{A} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where the process noise $\mathbf{W}$ has zero mean and suitable covariance matrix $\mathbf{Q}_{xy}$.

The algorithm used for particle filtering is the Sequential Importance Resampling (SIR) [4].

In our application, particle filter initialization is the major difficulty, because passive radar large measurement error makes tracking accuracy poor and because coordinates transformation from bi-static to Cartesian is ambiguous. In fact, while the correspondence between the bi-static pair $[r, \theta]$ and the Cartesian couple $[x, y]$ is univocal, the relation between Doppler frequency $f_d = -\frac{1}{\lambda}\frac{\partial r}{\partial t}$ and Cartesian velocity $[\dot{x}, \dot{y}]$ is ambiguous. It depends on the fact that Doppler shift only indicates how targets are moving with respect to the ellipse (whose foci coincide with receiver and transmitter of opportunity) they are currently lying on: if $\dot{r} > 0$, they are moving towards larger ellipses, and vice versa. Such information generates velocity uncertainty in the Cartesian domain, i.e. there are infinite pairs $[\dot{x}, \dot{y}]$ generating the same Doppler frequency.

Particle filter samples are therefore initialized according to a quasi deterministic rule for position $[x, y]$ and randomly for velocity $[\dot{x}, \dot{y}]$.

$$\begin{aligned} x &= x_{init} + n_x \\ y &= y_{init} + n_y \end{aligned} \tag{5}$$

In Equation (5), n_x and n_y are Gaussian white noises with zero mean and standard deviation Δ, while x_{init} and y_{init} are evaluated using Equation (6), that transforms the bi-static estimate $[\hat{r}, \hat{\theta}]$ into the Cartesian pair $[x, y]$.

$$\begin{cases} x = \dfrac{(\hat{r}+d)^2 - x_{Tx}^2 - y_{Tx}^2}{2(\hat{r}+d-x_{Tx}\cos\hat{\theta} - y_{Tx}\sin\hat{\theta})}\cos\hat{\theta} \\ y = \dfrac{(\hat{r}+d)^2 - x_{Tx}^2 - y_{Tx}^2}{2(\hat{r}+d-x_{Tx}\cos\hat{\theta} - y_{Tx}\sin\hat{\theta})}\sin\hat{\theta} \end{cases} \quad (6)$$

The velocity vector $[\dot{x}, \dot{y}]$ is instead generated randomly, according to Equation (7).

$$\begin{aligned} \dot{x} &= V\cos(\varphi) \\ \dot{y} &= V\sin(\varphi) \end{aligned} \quad (7)$$

Absolute value V is uniformly distributed in the velocity interval of interest for PCR applications; direction angle φ is uniformly distributed in $[0, 2\pi]$.

IV. Comparison between tracking algorithms

This section aims at evaluating and comparing the performance of three alternative filtering techniques to be used as Block 2 of Fig. 2:

1) geometric transformation;
2) Extended Kalman Filter;
3) Particle Filter.

The simple geometric transformation, given in Equation (6), is a low computational cost solution, estimating only Cartesian target position.

The three algorithms have been evaluated and compared by means of Monte Carlo simulations: on the purpose, 100 independent trials, with varying measurement noise, have been run. The metric for performance evaluation is the error in estimating the state of the simulated target. In particular, given the generic element $\xi(t)$ of the state vector at time t (true value) and the estimate $\hat{\xi}_i(t)$ associated to the i-th Monte Carlo trial, the computed metric is:

$$RMSE_{\xi}(t) = \sqrt{\frac{\sum_{i=1}^{100}\left(\hat{\xi}_i(t) - \xi(t)\right)^2}{100}} \quad (8)$$

where *RMSE* stands for Root Mean Square Error.

The metric $RMSE_{\xi}(t)$ provides the standard deviation of the estimate error at scan time t. Therefore, the mean value over the N_s radar scans considered in the simulation is:

$$\overline{RMSE}_{\xi} = \frac{1}{N_s}\sum_{t=1}^{N_s} RMSE_{\xi}(t) \quad (9)$$

The simulated scenario consists of one aircraft moving along a straight line with uniform speed for N_s=40 scans, as reported in Fig. 3, where the positions of the receiver (Rx) and of the FM transmitter (Tx) are indicated as well. The target initial state is:

$$\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 10km \\ 80km \\ 0.23km/s \\ 0km/s \end{bmatrix} \quad (10)$$

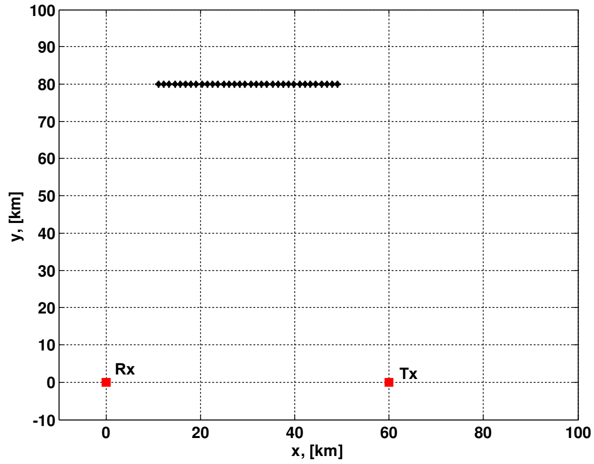

Fig. 3 Simulated target trajectory and bi-static geometry

Simulation parameters are reported in Table 1. Please note that process noise covariance matrix $\mathbf{Q}_{xy}$ is common to EKF and PF.

TABLE I
Simulation parameters

Measurement noise standard deviation	σ_r = 1.7 km σ_θ = 5 deg σ_{fd} = 1 Hz
Process noise covariance matrix for EKF and PF	$\mathbf{Q}_{xy}^{1/2}$= =diag([0.4km,0.4km, 0.04km/s, 0.04km/s])
Number of particles for PF	N_p = 10000
False alarm probability	P_{fa} = 0
Detection probability	P_d = 1
Receiver coordinates	**Rx** = [0km, 0km]
Transmitter coordinates	**Tx** = [60km, 0km]
Scan period	T = 5 s

Curves for metric $RMSE_{\xi}(t)$ are shown from Fig. 4 to Fig. 7, for each element of the state vector $\chi = [x, y, \dot{x}, \dot{y}]$. Resultant values of the metric $\overline{RMSE}_{\xi}$ are instead reported in Table 2.

RMSE curves show that PF performs better than EKF in estimating both position and velocity, from scan 1 to scan 20; successive scans (they are only 34 because the first 6 are necessary to produce confirmed tracks) are instead characterized by a substantial equivalency between the two tracking filters, which do anyway behave much better than the simple geometric transformation. The overlapping of the three accuracy curves at the first scan is due to the use of the output of the geometric transformation to initialize both EKF and PF. The overall improvement in tracking accuracy yielded by PF is definitely stated by the results reported in Table 2.

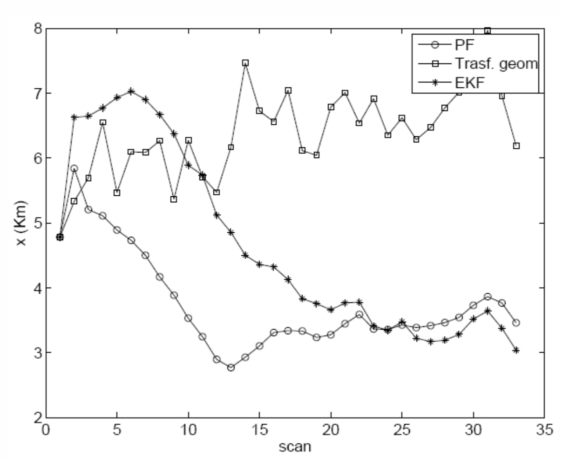

Fig. 4 *RMSE(t)* for coordinate *x*

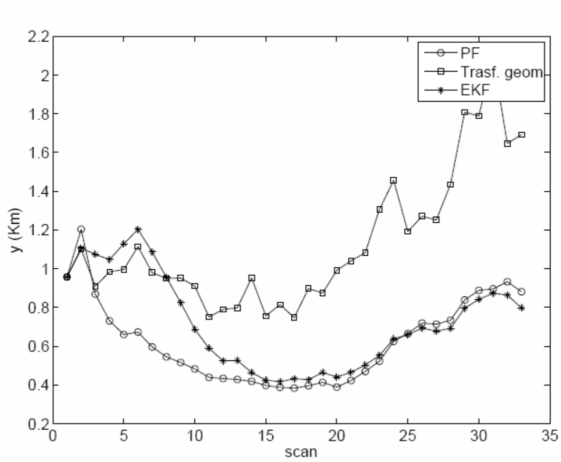

Fig. 5 *RMSE(t)* for coordinate *y*

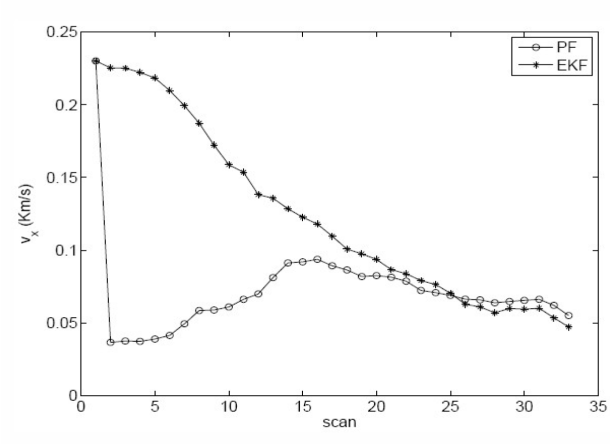

Fig. 6 *RMSE(t)* for coordinate v_x

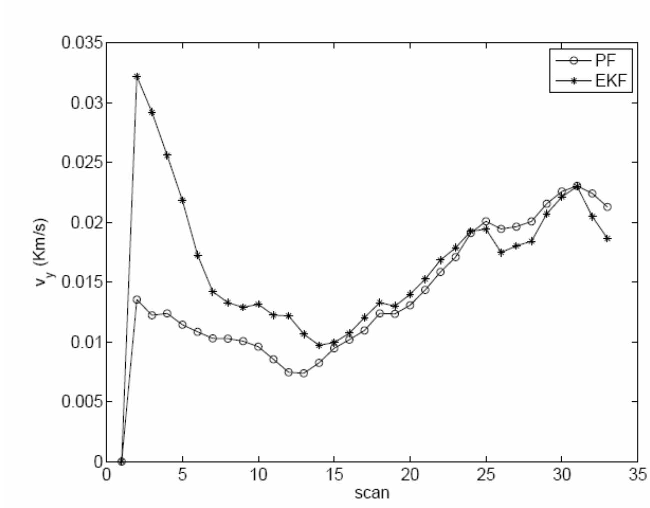

Fig. 7 *RMSE(t)* for coordinate v_y

TABLE II

VALUES OF $\overline{RMSE}$ METRIC

$\overline{RMSE}$	x (km)	y (km)	v_x (km/s)	v_y (km/s)
Geom.	6.4583	1.2713	-	-
EKF	4.7267	0.7744	0.1332	0.0181
PF	3.7914	0.6862	0.0771	0.00158

V. CONCLUSIONS

The innovative use of particle filtering in the data processing chain of a FM radio based passive radar has been validated through a set of Monte Carlo simulations. In particular, we have shown that:

- PF and EKF both perform much better than the simple geometric transformation of bistatic tracks into Cartesian tracks;
- PF converges more rapidly than EKF;
- PF has a computational load greater than EKF, but it widely respects real-time implementation requirements [1].

References

[1] A. Benavoli, A. Di Lallo, A. Farina, R. Fulcoli, R. Mancinelli, L. Timmoneri, L. Chisci, "Design and development of a signal and data processor test bed for a passive radar in the FM band", *Radar 2007*, Edinburgh, 15-18 October 2007.

[2] P.E. Howland, D. Maksimiuk, G. Reitsma. “FM radio based bistatic radar”, *IEE Proc. Radar Sonar Navigation*, vol. 152, pp. 107-115, 2005.

[3] B. Ristic, S. Arulampalam, N. Gordon. *Beyond the Kalman filter: particle filters for tracking applications*, Artech House, (2004).

[4] A. Doucet, N. De Freitas, N. Gordon, *Sequential Monte Carlo Methods in Practice*, Springer, 2001.